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AN INVESTIGATION OF THE STRUCTURAL ACTION
OF A PONY-TRUSS BRIDGE

by

ROBERT BRUCE PINKNEY, B.Sc.

A THESIS

SUBMITTED TO THE FACULTY OF GRADUATE STUDIES
IN PARTIAL FULFILMENT OF THE REQUIREMENTS
FOR THE DEGREE OF MASTER OF SCIENCE

DEPARTMENT OF CIVIL ENGINEERING

EDMONTON, ALBERTA
FEBRUARY, 1963

ABSTRACT

The structural action of a pony-truss bridge is exceedingly complex, since the only lateral support of the compression chords is provided by a U-shaped rigid frame at each panel point, comprising the vertical members and floor beam at that panel point. The present investigation was undertaken to attempt to discover some means whereby the capacity of a pony-truss bridge could be accurately assessed.

The method of slope-deflection has been applied by previous authors to the investigation of both the elastic stability of the compression chords and the secondary stresses produced during displacement of these members in a pony-truss bridge. The analysis developed by them has been presented in detail in this report.

A full-scale highway pony-truss bridge was static load-tested to collapse, using two different load distributions. In the first test, the entire load was applied to the center floor beam; in the second, the load was distributed among the center and two adjacent floor beams. The collapse loads were found to be, respectively, 45 tons and 65 tons for the two tests. The results of these tests, in conjunction with the slope-deflection buckling analysis, suggested that collapse might be initiated either by buckling of a compression chord, or by excessive secondary stresses.

The slope-deflection analysis was found to give theoretical lateral deflections of the top chords in fairly good agreement with those actually recorded. Thus the method appears promising for the computation of secondary stresses. Certain aspects of the analysis, however, could be improved.

There is ample scope for additional load tests, designed specifically to evaluate the accuracy of the analysis.

ACKNOWLEDGEMENTS

The author wishes to express his sincere appreciation

to Dr. J. S. Kennedy, Department of Mechanical Engineering,
University of Alberta, for his suggestions, criticisms and encouragement during every phase of this project;

to the Research Council of Alberta and the Bridge Branch of the
Department of Highways of the Province of Alberta, who, under the
administration of Mr. B. P. Shields, sponsored the testing program;

to Mr. Fred Vaneldik for his very capable assistance in preparing
for and carrying out the tests; and

to Mr. Barry Mailloux and members of the staff of the University of
Alberta Computing Center for their helpful suggestions in the preparation and running of the computer programs.

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NOTATION

a	fixity coefficient in bending of end joint
b	fixity coefficient in torsion of end joint
C	carry-over factor
E	modulus of elasticity
G	shear modulus
I	moment of inertia
J	torsional constant
K	bending stiffness of member with far end fixed
K'	modified stiffness of member in anti-symmetrical bending
L	length of member
M	moment acting on right end of member
M'	moment acting on left end of member
M_f	fixed-end moment in floor beam due to applied loads
m	rotation of right end of member about an axis perpendicular to axis of member and lying in plane of truss
m'	rotation of left end of member about an axis perpendicular to axis of member and lying in plane of truss
$n-1, n, n+1$	subscripts denoting joints or members
P	axial force in member
R	torsional rigidity of member
S	force at top of vertical due to bending of floor beam
S'	stiffness of cross-frame

S''	stiffness of diagonal
T	torsional moment
t	rotation of right end of member about its axis
t'	rotation of left end of member about its axis
α	rotation about x-axis
β	rotation about y-axis
δ	deflection along z-axis
δ'	initial eccentricity or initial deflection of a joint along the z-axis (relative to the vertical plane passing through the two end joints)
ζ	stiffness parameter
θ	angle between member and horizontal
ψ	stiffness parameter

CHAPTER I

STRUCTURAL ACTION OF THE PONY-TRUSS

The pony-truss bridge differs from common through highway bridges in that it lacks, for purposes of vehicle clearance, lateral bracing in the plane of the top, or compression, chords. Design of a compression member is predicated, of course, on its buckling tendency, which in turn is directly proportional to the square of its unsupported length, and inversely proportional to its moment of inertia about the axis of buckling. The omission of lateral bracing, then, increases the effective length of the top chord several times, depending on the number of panel points, thereby greatly reducing its resistance to buckling.

It is obvious that, to ensure stability, either the effective length of the member must be reduced by the introduction of lateral supports, or its moment of inertia must be increased. It has been found more economical to provide at least some measure of lateral support rather than to resort to inefficient stiffening of the top chord. This has been accomplished through the use of a U-shaped rigid frame (Figure 1.1), hereafter referred to as the cross-frame, consisting of the floor beam and the two vertical members at each panel point of the bridge. The cross-frame is designed with a so-called rigid connection between the verticals and the floor beam.

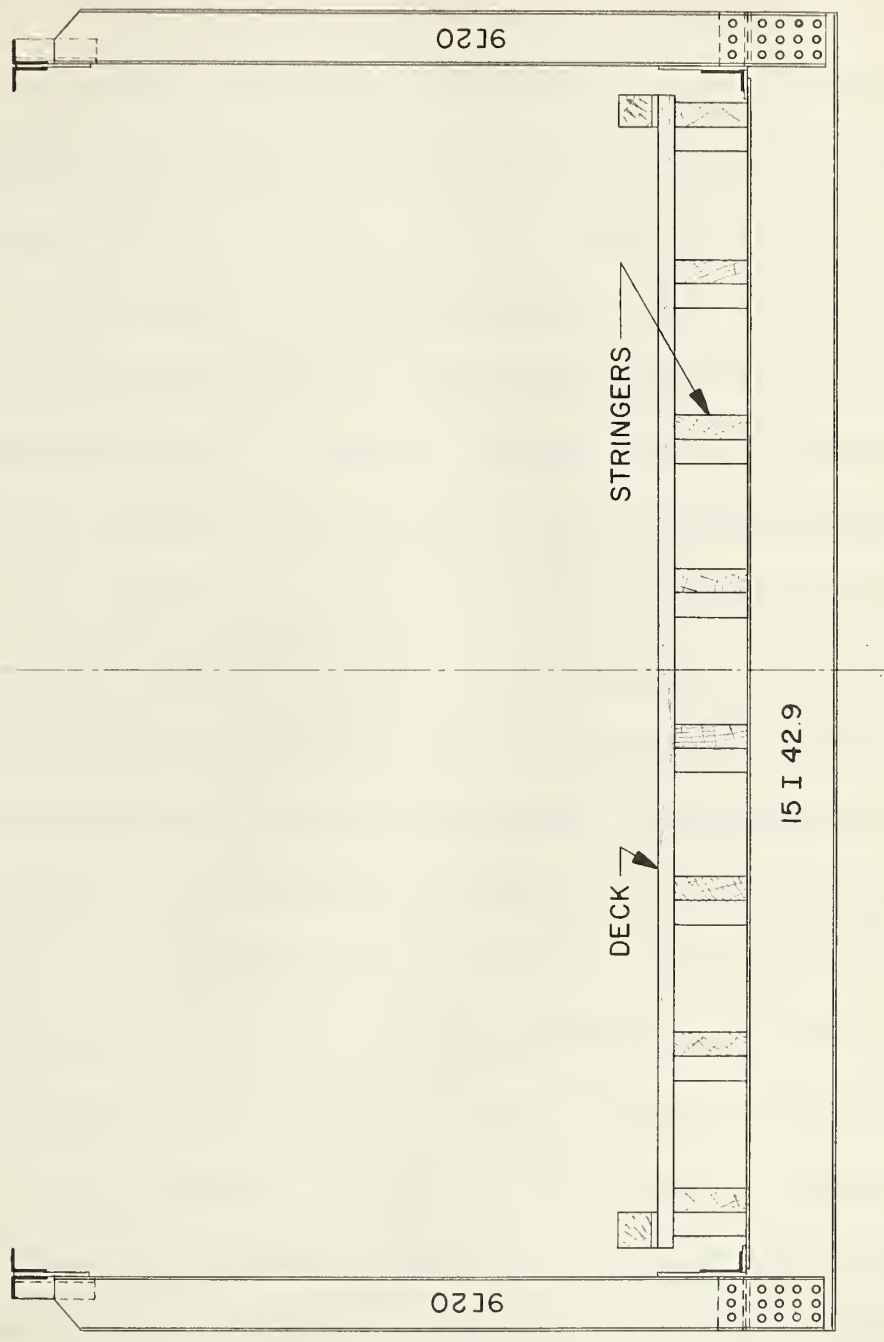


FIGURE I.1 - CROSS-FRAME OF TEST BRIDGE

The stiffness of the cross-frame, regarded here as its capacity to resist deformation under a given lateral load at the top of the vertical, becomes a decisive factor in the determination of the load-carrying capacity of the bridge. In general, increased stiffness provides increased lateral support for the top chord, both through its resistance to lateral deflection and its resistance to torsional deformation of the top chord. In the simplest case, where all of the floor beams are equally loaded, the problem approaches the more basic theoretical one of the buckling of a continuous column on elastic supports, the cross-frame stiffness being a measure of the spring constant.

Practically, however, there are several complicating factors. As has been indicated, differential beam loading, and hence differential cross-frame deformation, causes a change in the relative degree of support offered to the top chord by each cross-frame; that is to say, the zero-lateral-load position of the top of a given vertical member changes as the attached floor-beam deflects under applied load. It is entirely possible that certain loading conditions can actually cause the cross-frame to assist rather than resist buckling tendencies. A further effect, due more to the action of the verticals themselves than to that of the cross-frame, involves their torsional stiffness. Since buckling of the top chord is accompanied by rotation about the axis of the verticals, the greater their torsional stiffness, the greater their assistance in resisting buckling.

In addition to the actions of the cross-frame and verticals, other web members in a similar fashion exert an influence on the behavior of the top chord, through their bending and torsional stiffnesses. Their lower chord support conditions, however, are more difficult to assess, and they are not directly affected by deformation of the floor beams.

Another secondary effect arises from the fact that, due to the nature of the connection of the top chord to the web members, the top chord itself is forced to undergo torsional deformation during differential lateral deflection of panel points. Thus the torsional stiffness of the top chord can affect the buckling tendency.

The presence of axial, or direct stress in a member modifies both its bending and torsional stiffnesses.

Although there are other factors, mentioned briefly later, it is felt that the above are of greatest significance in considering the stability of the pony-truss. Their relative importance is discussed in connection with the analysis to be described in this report.

I. THE PRESENT PROBLEM

During the early part of this century, around 1920, pony-truss bridges were being erected in the Province of Alberta as Standard Light Highway Spans on rural highways. These bridges were probably overdesigned with regard to any traffic loads that might at that time be

imposed upon them, but their ultimate capacity can now be easily exceeded by the loads attained by modern heavy transport vehicles. In fact, the Department of Highways of the Province of Alberta has on record descriptions of the collapses of several such bridges.

The current interest in the pony-truss arises from the fact that many of these old bridges are still in service in Alberta. Furthermore, when old bridges are replaced by new ones, the old are dismantled and re-erected in less demanding locations. Thus any decrease in the total number of pony-trusses will be small in the foreseeable future.

This study was undertaken to try to provide sufficient information to enable bridges to be load-rated more accurately than previously, and possibly to suggest methods of strengthening.

II. THIS REPORT

The first part of this report is concerned with the theoretical analysis of a pony-truss, in particular the pony-truss used in the testing program. The method of analysis is described in detail in the following chapter.

The remainder is devoted to a description of the actual testing, and the presentation and discussion of test results, with an attempt to relate the results of the analysis to the observed behavior and failure loads of the bridge.

III. BACKGROUND OF THE ANALYSIS

The problem of the analysis and design of a pony-truss bridge, or the more basic problem of the buckling of a continuous beam-column on elastic supports, has for many years been the subject of discussion and research by numerous authors. One of the more recent of these investigators, Edward C. Holt, Jr., has compiled what is probably the most comprehensive bibliographical treatment available. His paper ^{(1)*}, published in 1952, was the first of a series entitled "The Stability of Bridge Chords Without Lateral Bracing", sponsored by the Column Research Council of Engineering Foundation, the Pennsylvania State Highway Department and the U.S. Bureau of Public Roads.

In this first report, Holt discusses only the fundamental problem of the buckling of a straight chord supported at intervals by transverse springs. He considers the theories advanced by various previous investigators, categorizing them according to methods of analysis and basic assumptions used. The different methods may be summarized briefly:

1. The continuous distribution of the elastic support along the chord is assumed, reducing the problem to that of a continuous beam-column on an elastic foundation.
2. Using standard methods of indeterminate analysis (e.g. slope-deflection, virtual work, three moment equation), simultaneous, linear, homogeneous equations may be set up, by means of

*Numbers in parentheses refer to references listed in the Bibliography.

which the buckling load can be determined.

3. A set of simultaneous equations such as those mentioned above can be reduced, through certain simplifying assumptions, to a set of finite difference equations. These may be solved for the required cross-frame stiffnesses.
4. Energy methods can be used to solve the problem, but require the assumption of a buckled shape for the chord.
5. Relaxation and numerical procedures can be employed.

Holt then analyzed a typical highway truss at a given working load, according to the procedures developed by Bleich, Engesser, Timoshenko and others. The results were presented in the form of safety factors against buckling. It is significant to note that only minor differences appeared in the final factors of safety, the range of variation being about five per cent in a safety factor of about 2.2.

The second report⁽²⁾ of the series, also by Holt, presents an analytical investigation of a number of secondary effects, some of which have already been discussed, and their influence on the computed safety factor:

1. the bending and torsional stiffnesses of the diagonals;
2. the torsional stiffness of the verticals;
3. the direct stress in the verticals;
4. the torsional stiffnesses of the top chord and end posts;
5. using the developed length of the chord and end posts;
6. neglecting bending in the end posts; and
7. chord curvature, that is, a non-parallel-chorded truss.

The results of this investigation are shown in Table 1.1:

TABLE 1.1

Chord Type	Effect Neglected	Factor of Safety	% Error
Str.	None	2.283	-----
Str.	Diagonals (1)*	2.261	+0.96
Str.	Stresses in Web (2 & 3)	2.264	+0.83
Str.	Chord Torsion (4)	2.226	+2.50
Str.	End Post Slope (5 & 6)	2.264	+0.83
Curved	None	2.282	-----
Curved	Chord Curvature (7)	2.284	-0.09

*numbers in parentheses refer to numbered list in preceding paragraph.

Each effect neglected is in addition to those preceding it in the table, and each per cent error is due to the cumulative effects neglected. It will be noticed that the errors have been compensating to some extent. Positive errors are on the safe side.

It should be noted that Holt points out that the conclusions of Table 1.1 are valid only for steel structures which fail in the inelastic range. For other materials, or for structures failing elastically, he states that substantial errors may arise through neglect of the secondary effects enumerated.

Two additional papers by Holt, published in 1956 and 1957 as Reports 3⁽³⁾ and 4⁽⁴⁾, respectively, of the same series, present the results of tests on a total of ten model pony-trusses. These bridges failed at an average of 13.6% less than the theoretical buckling loads computed by Holt, and within a range of from 7% to 17% less.

The analysis used in the papers consisted of equating to zero, through trial-and-error, the stiffness of the upper chord joints. The stiffness of a joint is the force-displacement relationship of the joint, comprising the effects of the stiffnesses of all members framing into the joint. When the stiffness of any joint approaches zero, a small increase in load produces a very large displacement; the load at which this phenomenon occurs is considered as the theoretical buckling load. The analysis also provides for the computation of the stresses arising from the secondary effects previously mentioned.

Holt concludes these papers with certain recommendations for the design of pony-truss bridges, paraphrased here:

Design of Top Chords: The design of the top chords should be based on a buckling analysis. He suggests using an "effective length factor", found by a buckling analysis, in the usual specification for the design of compression members.

Design of End Posts: Each end post should be designed as a cantilever to carry in addition to its axial load, the bending moment which would be produced by a transverse force of 0.3% of the computed axial load, applied at the upper end. Design for buckling should be based on an effective length factor similar to that of the top chord.

Design of Verticals: Each vertical should be designed as a cantilever with a transverse force of 0.2% of the average of the axial loads in the two adjacent top chord members, applied at the upper end.

Design of Floor-Beam Connection: The connection of the vertical to the floor-beam should be designed to develop the full capacity of the vertical.

In 1958, S. L. Lee and R. W. Clough, Jr., published a paper⁽⁵⁾

taken from a doctoral thesis by the former. This paper presents a pony-truss analysis based on the method of slope-deflection. A group of simultaneous equations of equilibrium may be set up for a given load, wherein certain moments and forces exerted on an upper chord panel point by chord and web members are equated to the corresponding resisting moments and forces exerted by the cross-frame. These equations can then be expressed in terms of rectilinear and angular displacements through the use of appropriate stiffness factors, with the joint displacements as unknowns on the left side of the equations. The theoretical buckling load can then be computed by setting the determinant of the coefficients equal to zero (by bracketing and interpolating values for several loads), since at this point the unknowns (that is, the displacements) become very large. It is possible, however, that some members of the truss may yield at a load lower than the theoretical buckling load. In order to predict this, it is necessary to know the bending stresses in the members. These may be determined for loads other than the buckling load by first computing the unknown displacements, then in turn using these, along with stiffnesses, to compute bending moments. If the computed stresses indicate that a given member has yielded, then collapse is a possibility. Inspection of the truss should disclose whether or not the member is required for stability.

Although this paper makes no mention of any experimental

verification of the theory, the authors have analyzed, as an example, the same truss analyzed by Holt in his first two papers.^{(1), (2)} The safety factors found by the two analyses differed by only about 0.5%. For this reason, and because of the easier accessibility of the paper by Lee and Clough, this method has been used in the present study, and is discussed more fully in the succeeding chapter.

CHAPTER II

SLOPE-DEFLECTION ANALYSIS OF A PONY-TRUSS BRIDGE

The analysis presented in this chapter was developed by Lee and Clough, Jr., and may be found in their paper entitled "Stability of Pony-Truss Bridges".⁽⁵⁾

The analysis is an application of the so-called Method of Slope-Deflection, whereby certain forces and couples acting upon the upper chord joints are expressed in terms of related displacements, and the equations of statics applied to each joint. The resulting set of simultaneous linear equations may then be solved for the displacements.

I. PRIMARY CONSIDERATIONS

Any joint in a truss has, in general, six degrees of freedom. It may translate along or rotate about any of three mutually perpendicular axes. For the purpose of this analysis, a system of coordinates has been devised as shown in Figure 2.1, using any given joint as the origin. The truss therefore lies in the x-y plane, and the z-axis is perpendicular to it.

If a load is applied to, and in the plane of, the truss, there will occur translational displacements in the x- and y-directions, and rotational displacements about the z-axis. In the case of a pony-truss bridge, however, the load is not applied in the plane of the truss, and

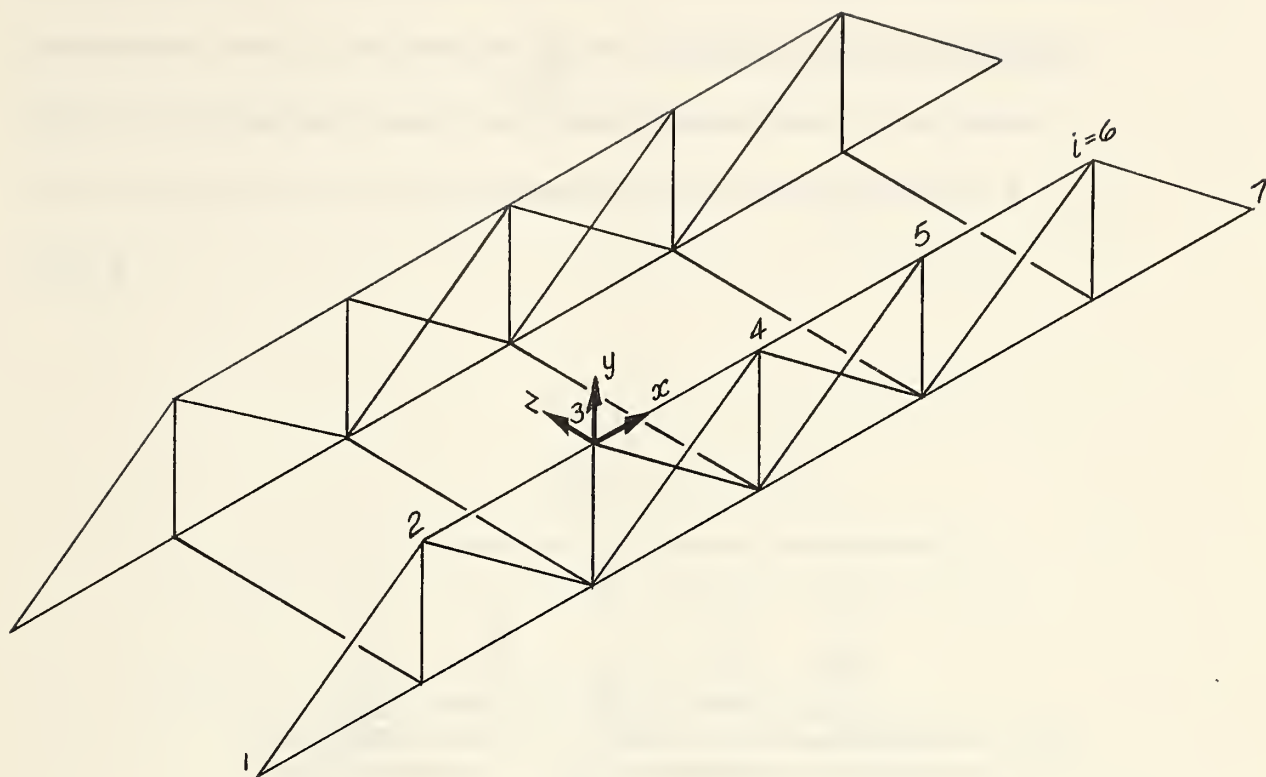


Figure 2.1

the vertical members are tilted by virtue of their interaction with the floor beams. Thus the top chord joints are displaced out of the plane of the truss. Examination of Figure 2.2 will indicate that this displacement may be conveniently resolved into three components, translation in the z -direction and rotation about the x - and y -axes.

Provided that the truss members are symmetrical with respect to the plane of the truss, the three displacement components associated with plane deformation do not influence the lateral

displacements of the top chord joints. Thus only three displacement components need be considered in the analysis of lateral stability: deflection along the z-axis, and rotation about the x- and y-axes. These displacements will hereafter be denoted respectively δ , α , and β .

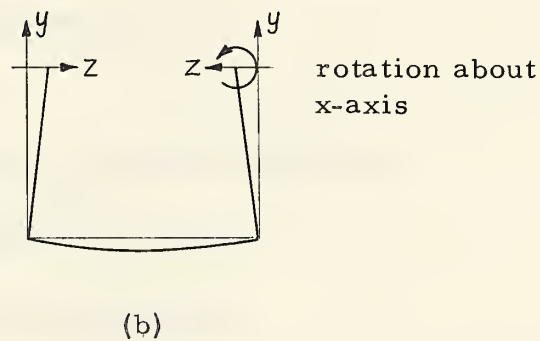
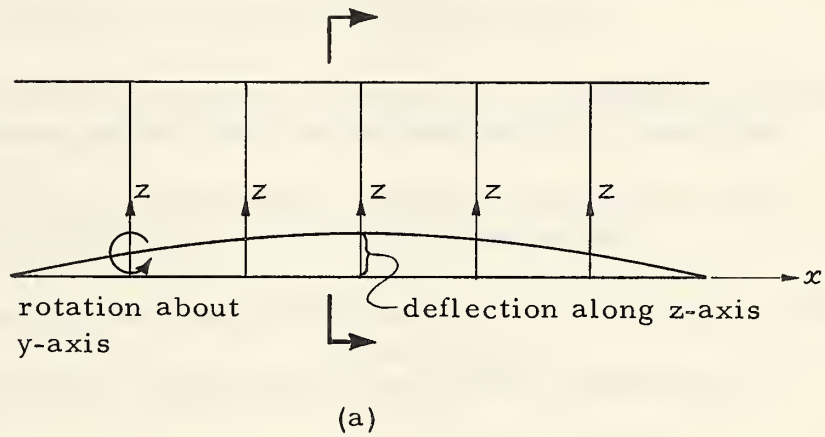


Figure 2.2

Where a member is unsymmetrical, as in the test span, bending is accompanied by torsional displacement, and rectilinear displacements in the plane of the truss do influence the lateral

displacements of the top chord joints. If, however, the torsional stiffness of such a member is small, a large displacement will be required to produce an appreciable torsional moment. Since the unsymmetrical members in the experimental truss comprised only light angles and channels, it was felt that the lack of symmetry could be neglected.

Further assumptions made by Lee in the analysis include:

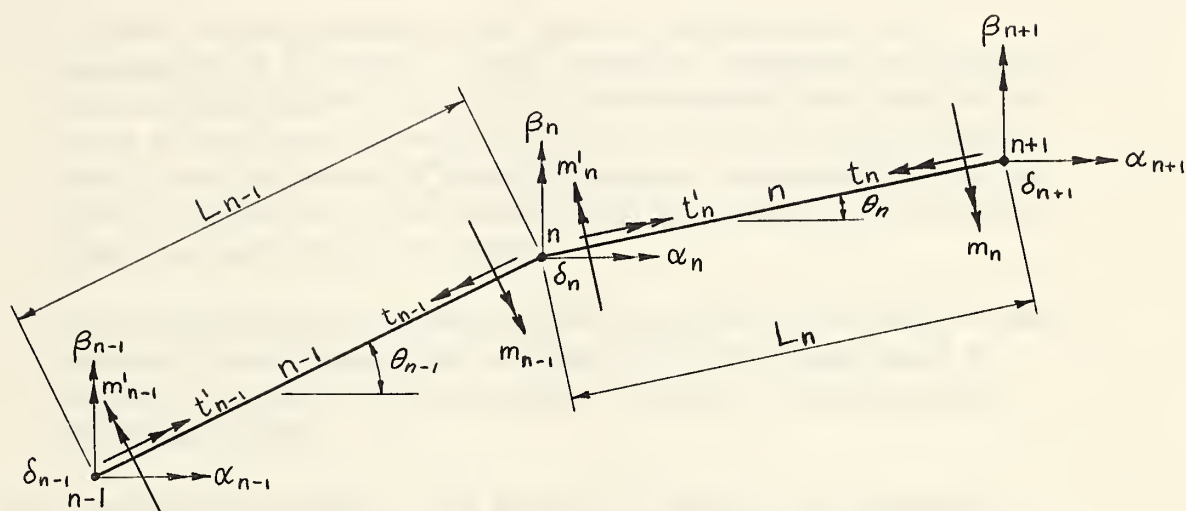
1. The diagonals are fixed at their lower ends.
2. The verticals are fixed in torsion, but elastically supported in bending by the floor beams forming the cross-frame.
3. The influence of the bottom chords and diagonals on the bending of the cross-frame may be neglected.

The analysis has been developed to take into account a number of secondary effects, stated here as set forth in Reference 3:

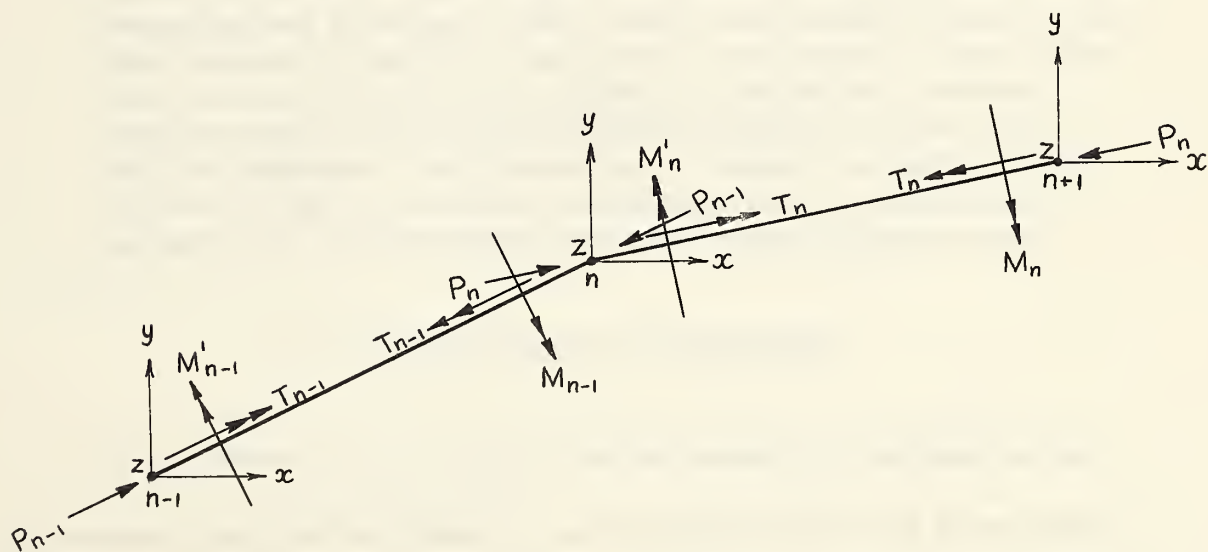
1. Varying axial compression along the top chord;
2. Non-uniform chord sections;
3. Chord curvature;
4. Various support conditions of the end posts;
5. Torsional stiffness of the members;
6. Support provided by the diagonals;
7. Axial thrust in the web members;
8. Continuity between chord and web members;
9. Unequal joint stiffnesses;
10. Bending of floor beams; and

11. Initial eccentricities.

The steps involved in the derivation of the slope-deflection equations are as follows, referring to Figure 2.3:



(a) Displacements



(b) Forces & Moments

Figure 2.3

1. A reference system is set up for any given joint " n ". The adjacent joints may then be designated " $n-1$ " and " $n+1$ ". Each joint is the origin of three mutually perpendicular axes, with their positive directions as shown in Figure 2.1. The chord member immediately to the right of a joint is designated by the same number as the joint.
2. Certain displacements of the ends of each member are designated as in Figure 2.3(a), namely, rotation " m " about an axis perpendicular to the axis of the member and lying in the plane of the truss, rotation " t " about the axis of the member itself, and deflection " δ " along the z-axis, perpendicular to the plane of the truss. The displacements of the joints about the principal axes are also shown.
3. The bending and torsional moments applied to the ends of each member, also designated in Figure 2.3, are expressed in terms of the above displacements and their corresponding stiffness factors.
4. These displacements, and hence the bending and torsional moments, are expressed in terms of the joint displacements along or about the principal coordinate axes.
5. Three equations of equilibrium are established for each joint by summing and equating to zero, the forces along the z-axis, and the moments about the x- and y-axes respectively. Each joint must remain in equilibrium under the forces and moments exerted upon it by the top chord members, the cross-frame, and the diagonals. All forces and moments, including those exerted by the cross-frame and diagonals, are expressed in terms of the corresponding stiffnesses and displacements.

II. DERIVATION OF EQUATIONS

The stiffness factor K of a flexural member is defined as the end moment required to produce a unit rotation of the end of the member to which it is applied, if the opposite end remains fixed. The carry-over factor C is the fraction of the moment K which is thereby induced at the fixed end of the member.

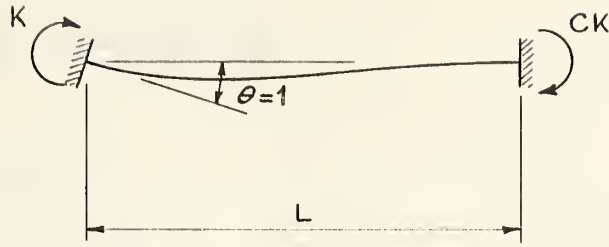


Figure 2.4

Expressions for K and C have been developed⁽⁶⁾ for members subjected to bending and axial load and are given here:

$$K = \frac{EI}{L} \left[\frac{12\psi}{4\psi^2 - \zeta^2} \right] \quad 1$$

$$C = \frac{\zeta}{2\psi} \quad 2$$

where E = modulus of elasticity

I = moment of inertia about the axis of bending

L = length of the member.

For compression members

$$\zeta = \frac{6(\phi \csc \phi - 1)}{\phi^2} \quad 3$$

$$\psi = \frac{3(1 - \phi \cot \phi)}{\phi^2} \quad 4$$

For tension members,

$$\zeta = \frac{6(1 - \phi \operatorname{csch} \phi)}{\phi^2} \quad 5$$

$$\psi = \frac{3(\phi \coth \phi - 1)}{\phi^2} \quad 6$$

In these expressions

$$\phi = L \sqrt{\frac{P}{EI}} \quad 7$$

where P is the absolute value of the axial force in the member.

The modified stiffness K' for antisymmetrical bending is defined as the moment required at each end of a member to produce a unit rotation at each end in the same direction. It may be expressed in terms of the stiffness factor K :

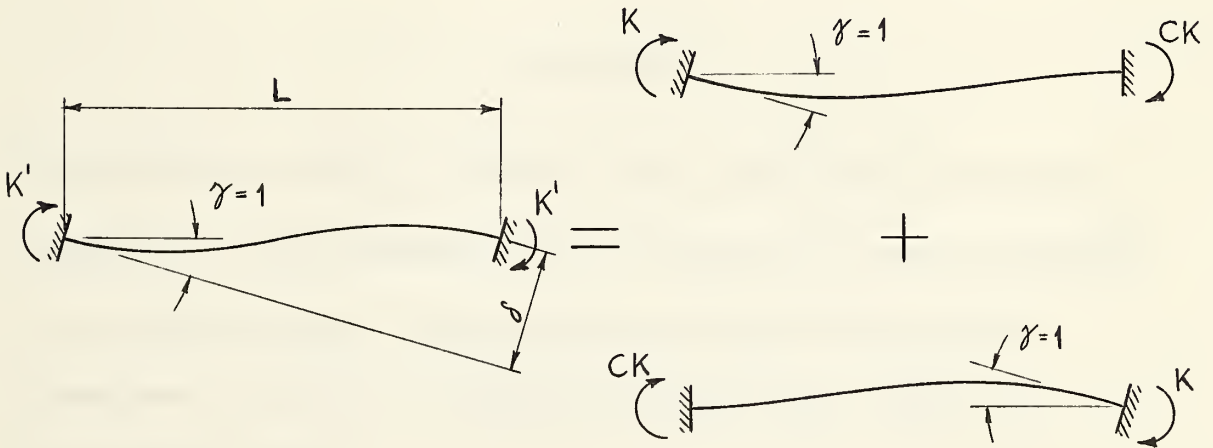


Figure 2.5

From Figure 2.5,

$$K' = K + CK = K(1 + C)$$

8

For small angles,

$$\gamma = \frac{\delta}{L}$$

so that the stiffness factor for a unit relative deflection δ , is equal to $\frac{K'}{L}$.

Consider now the general case of bending of a single upper chord member "n". Figure 2.6 shows the end moments and displacements developed.

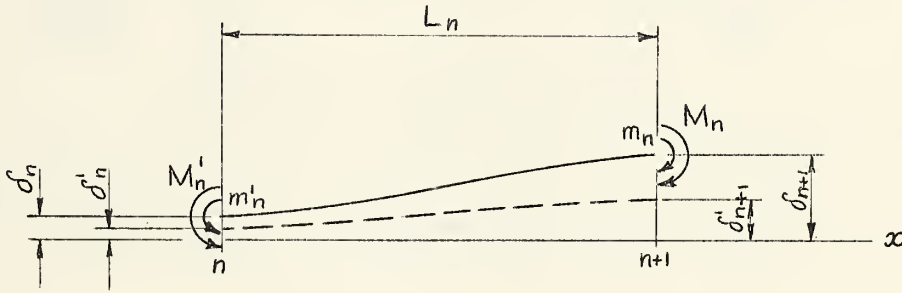


Figure 2.6

Note that the directions of rotations and couples shown correspond to Figure 2.3. The end moments shown in Figure 2.6 may be separated into components due to rotation of each joint and to differential deflections of the joints. Consider the moment M_n ; Figure 2.7.

From Figure 2.7(c),

$$\begin{aligned} \gamma &= \gamma'' - \gamma' \\ &= \frac{\delta_{n+1} - \delta_n}{L_n} - \frac{\delta'_{n+1} - \delta'_n}{L_n} \end{aligned}$$

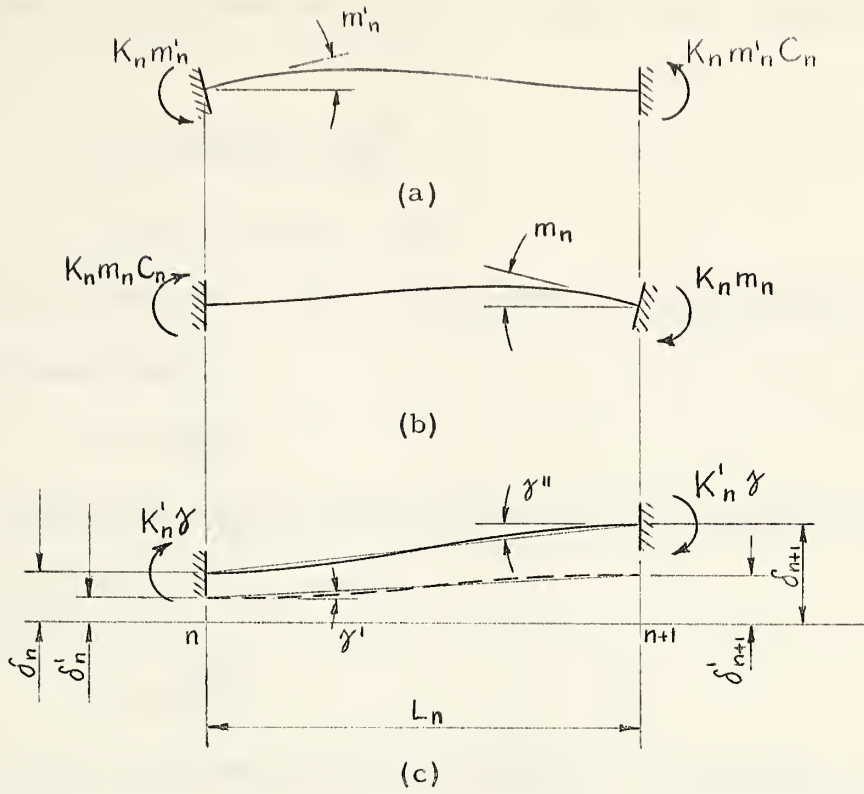


Figure 2.7

and

$$M_n = -K_n m'_n C_n + K_n m_n + \frac{K'_n}{L_n} (\delta_{n+1} - \delta_n) - \frac{K'_n}{L_n} (\delta'_{n+1} - \delta'_n) \quad 9$$

Similarly,

$$M'_n = -(\delta_{n+1} - \delta_n) \frac{K'_n}{L_n} - K_n C_n m_n + K_n m'_n + (\delta'_{n+1} - \delta'_n) \frac{K'_n}{L_n} \quad 10$$

$$M_{n-1} = (\delta_n - \delta_{n-1}) \frac{K'_{n-1}}{L_{n-1}} + K_{n-1} m_{n-1} - K_{n-1} C_{n-1} m'_{n-1} - (\delta'_n - \delta'_{n-1}) \frac{K'_{n-1}}{L_{n-1}} \quad 11$$

$$M'_{n-1} = -(\delta_n - \delta_{n-1}) \frac{K'_{n-1}}{L_{n-1}} - K_{n-1} C_{n-1} m_{n-1} + K_{n-1} m'_{n-1} + (\delta'_n - \delta'_{n-1}) \frac{K'_{n-1}}{L_{n-1}} \quad 12$$

By definition, the torsional moments T in adjacent top chord members are

$$T_n = R_n(t_n + t'_n) \quad 13$$

$$T_{n-1} = R_{n-1}(t_{n-1} + t'_{n-1}) \quad 14$$

where R is the torsional rigidity of the member. For members carrying no axial load,

$$R = \frac{GJ}{L}$$

where G is the shear modulus and J is the torsional constant of the cross-section. For open sections,

$$J = \frac{\sum bt^3}{3}$$

where " b " and " t " are the breadth and thickness, respectively, of rectangular elements of the section⁽⁷⁾.

The rotational displacements of the members shown in Figure 2.3 are now expressed in terms of the displacements of the joint about the principal axes.

$$m'_{n-1} = -\alpha_{n-1} \sin \theta_{n-1} + \beta_{n-1} \cos \theta_{n-1} \quad 15(a)$$

$$m_{n-1} = \alpha_n \sin \theta_{n-1} - \beta_n \cos \theta_{n-1} \quad 15(b)$$

$$m'_n = -\alpha_n \sin \theta_n + \beta_n \cos \theta_n \quad 15(c)$$

$$m_n = \alpha_{n+1} \sin \theta_n - \beta_{n+1} \cos \theta_n \quad 15(d)$$

$$t'_{n-1} = \alpha_{n-1} \cos \theta_{n-1} + \beta_{n-1} \sin \theta_{n-1} \quad 15(e)$$

$$t_{n-1} = -\alpha_n \cos \theta_{n-1} - \beta_n \sin \theta_{n-1} \quad 15(f)$$

$$t'_n = \alpha_n \cos \theta_n + \beta_n \sin \theta_n \quad 15(g)$$

$$t_n = -\alpha_{n+1} \cos \theta_n - \beta_{n+1} \sin \theta_n \quad 15(h)$$

The cross-frame and diagonals framing into each joint produce the forces and moments required to maintain equilibrium of that joint. Expressions may now be developed for these forces and moments.

Bending of the floor beams. Assuming the tops of the verticals to be fixed, bending of a floor beam develops a lateral force at the top of the vertical member, and a moment about the axis of the top chord, as indicated in Figure 2.8. No moment is developed, however, about the vertical axis. Expressions for the force and moment, designated respectively as S_z and S_x , are developed below. The load "w" is any symmetrically distributed load.

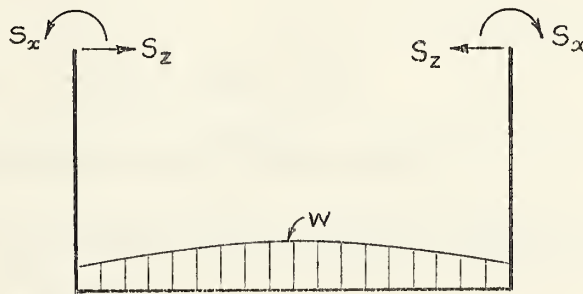


Figure 2.8

Let M_f represent the fixed-end moment in the floor beam due to

the applied load. Since the frame and loading are symmetrical, the stiffness factor of the floor beam may be modified for symmetrical bending, and only one-half of the frame analyzed, using the method of moment-distribution.

The modified stiffness K'' of a member in symmetrical bending is defined as the moment which must be applied to each end of the member in order to produce a unit rotation at each end in opposite directions, and is developed below.

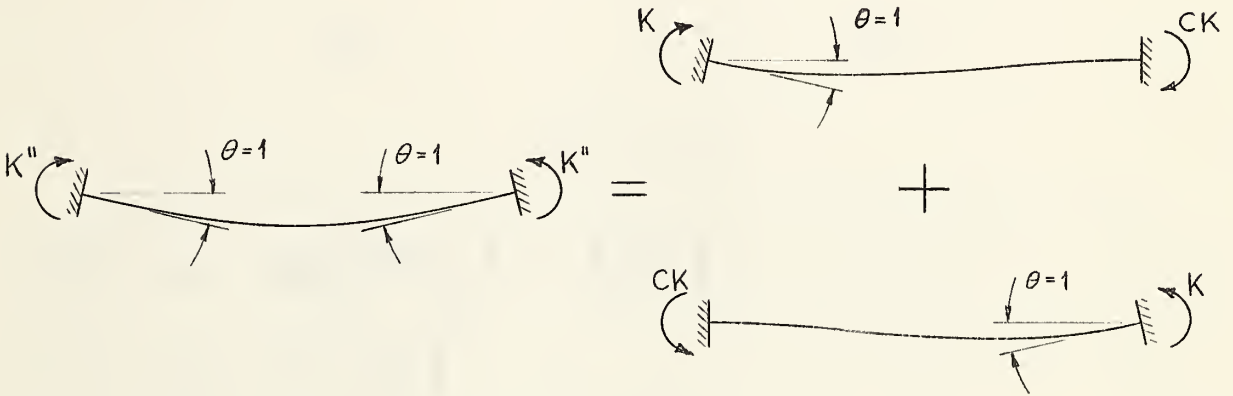


Figure 2.9

Thus

$$K'' = K - CK = K(1 - C)$$

16

where K is the stiffness factor of the member.

One-half of the frame is now analyzed (Figure 2.10), using the rigid frame sign convention.

Let
$$r = \frac{K''_b}{K_v}$$

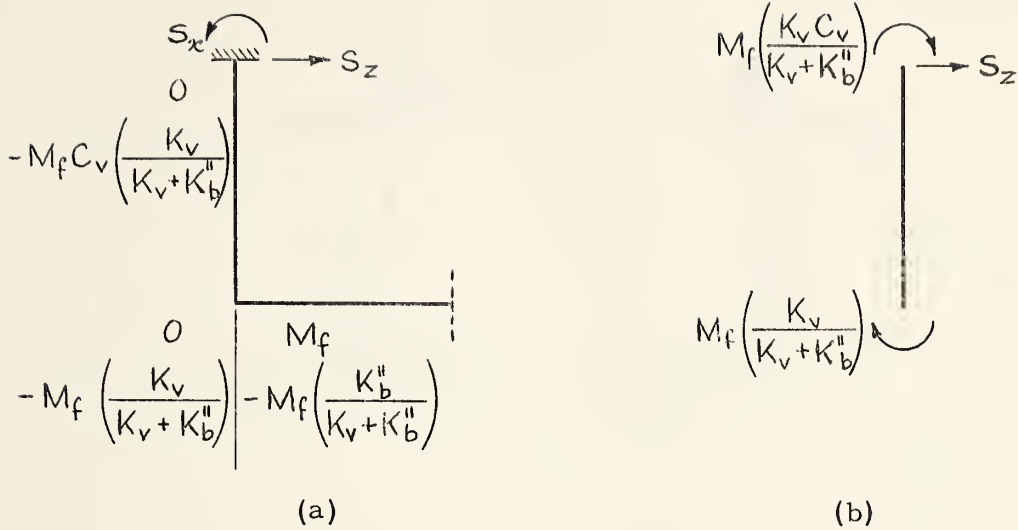


Figure 2.10

$$\text{Then } S_x = -M_f \left(\frac{K_v C_v}{K_v + K_b''} \right) = -M_f \left(\frac{C_v}{1 + r} \right) \quad 17$$

$$\text{and } S_z = -M_f \left(\frac{1 + C_v}{1 + r} \right) \quad 18$$

Note that when used in these equations, the fixed-end moment is positive when producing concave downward bending of the beam.

Stiffnesses of the cross-frame. The stiffnesses of the cross-frame may be similarly developed. A system of double subscripting is used which is best explained by an example: S'_{xz} is defined as the moment induced about the x-axis by a unit deflection along the z-axis, or as the force induced in the z-direction by a unit rotation about the x-axis. Figure 2.11 illustrates two stiffness factors, S'_{xz} and S'_{zz} .

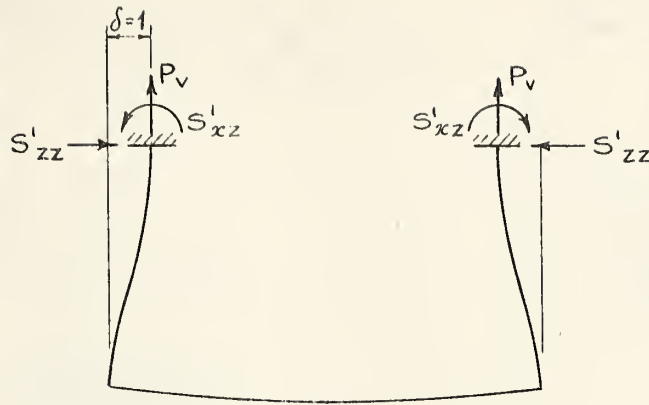


Figure 2.11

The stiffnesses shown above may be evaluated by distributing the fixed-end moments produced in the verticals by a unit deflection. From Figure 2.5, it is observed that the fixed-end moment produced by a deflection δ may be represented as

$$M_f = K' \frac{\delta}{L} \quad 19$$

For a unit deflection of the top of the vertical, then, Equation 19 becomes

$$M_f = \frac{K'_v}{L_v}$$

Again using the modified stiffness factor for symmetrical bending of the floor beam, it is necessary to analyze only one-half of the cross-frame.

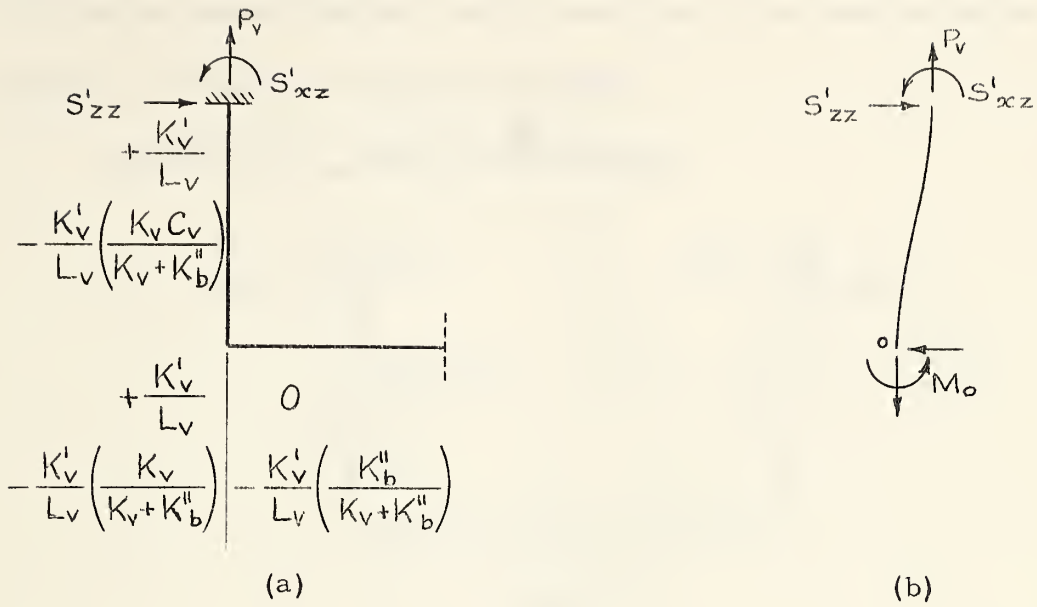


Figure 2.12

Letting $r = \frac{K''_b}{K_v}$ as previously,

then

$$\begin{aligned}
 S'_{xz} &= \frac{K'_v}{L_v} - \frac{K'_v}{L_v} \left(\frac{C_v}{1+r} \right) \\
 &= \frac{K'_v}{L_v} \left(1 - \frac{C_v}{1+r} \right)
 \end{aligned}$$

20

and, summing moments about point O,

$$\begin{aligned}
 S'_{zz} &= \frac{1}{L_v} \left[S'_{xz} + M_o + P_v \times l \right] \\
 &= \frac{1}{L_v} \left[\frac{K'_v}{L_v} \left(1 - \frac{C_v}{1+r} \right) + \frac{K'_v}{L_v} \left(1 - \frac{1}{1+r} \right) + P_v \right] \\
 &= \frac{K'_v}{L_v^2} \left(2 - \frac{1+C_v}{1+r} \right) + \frac{P_v}{L_v}
 \end{aligned}$$

21

It should be pointed out that in this equation, P_v is positive when tension, and negative when compression.

Similarly, S'_{xx} may be determined.

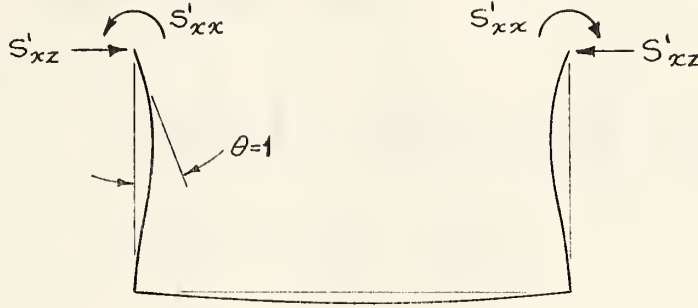
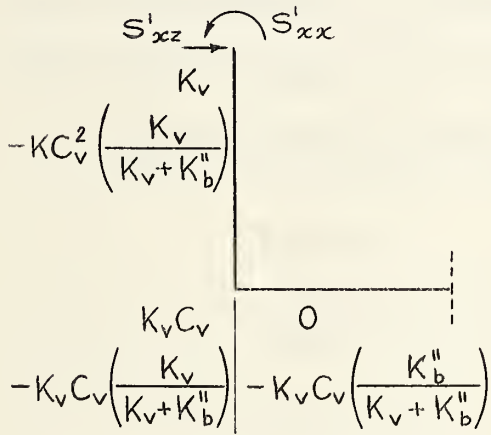
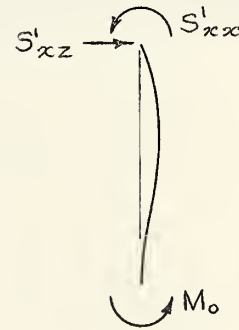


Figure 2.13

Figure 2.14(a) shows the distribution of fixed-end moments.



(a)



(b)

Figure 2.14

Therefore
$$S'_{xx} = K_v - K_v C_v^2 \left(\frac{K_v}{K_v + K''_b} \right)$$

But
$$K_v = \frac{K'_v}{1 + C_v} \quad \text{(from Equation 8)}$$

and
$$r = \frac{K''_b}{K_v}$$

Thus
$$S'_{xx} = K_v \left(1 - \frac{C_v^2}{1 + r} \right)$$
 22

and
$$\begin{aligned} S'_{xz} &= \frac{1}{L_v} (S'_{xx} + M_o) \\ &= \frac{1}{L_v} \left[K_v \left(1 - \frac{C_v^2}{1 + r} \right) + K_v C_v \left(1 - \frac{1}{1 + r} \right) \right] \\ &= \frac{K_v}{L_v} (1 + C_v) \left[1 - \frac{C_v}{1 + r} \right] \\ &= \frac{K'_v}{L_v} \left(1 - \frac{C_v}{1 + r} \right) \end{aligned}$$

It will be noticed that the force stiffness expressed by this last equation is identical to the moment stiffness expressed by Equation 20, confirming a previous statement to that effect.

Of the remaining stiffnesses,

$$S'_{xy} = 0$$

$$S'_{yz} = 0$$

$$S'_{yy} = R_v, \quad 23$$

each of which may be established by inspection, remembering the assumed conditions of symmetry.

Stiffnesses of the diagonals. The stiffnesses of the diagonals are more easily developed, since it has been assumed that they are

fixed at the bottom chord. The same system of double subscripting is used as for the stiffnesses of the cross-frame. Certain of these stiffnesses are shown in Figure 2.15.

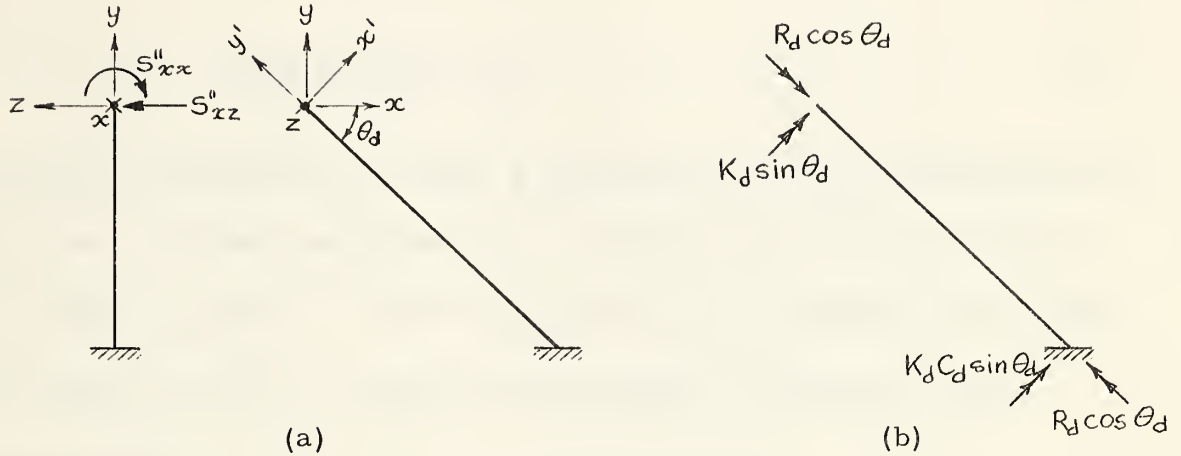


Figure 2.15

To determine S''_{xx} and S''_{xy} , a unit rotation is applied about the x-axis. The corresponding rotations about the x' - and y' -axes are, respectively, $1 \times \sin \theta_d$ and $1 \times \cos \theta_d$. The bending and torsional moments induced are, respectively, $K_d \sin \theta_d$ and $R_d \cos \theta_d$, as shown in Figure 2.15(b). These moments may be resolved into their x- and y-components:

$$S''_{xx} = K_d \sin^2 \theta_d + R_d \cos^2 \theta_d \quad 24$$

and
$$S''_{xy} = (K_d - R_d) \sin \theta_d \cos \theta_d \quad 25$$

Also,
$$S''_{xz} = \frac{K_d \sin \theta_d + K_d C_d \sin \theta_d}{L_d}$$

$$= \frac{K_d}{L_d} (1 + C_d) \sin \theta_d = \frac{K'_d}{L_d} \sin \theta_d \quad 26$$

The remaining stiffnesses, similarly determined, are as follows:

$$S''_{yy} = K_d \cos^2 \theta_d + R_d \sin^2 \theta_d \quad 27$$

$$S''_{yz} = \frac{K'_d}{L_d} \cos \theta_d \quad 28$$

$$S''_{zz} = \frac{2K'_d}{L_d^2} + \frac{P_d}{L_d} \quad 29$$

In the above equations, the angle θ is measured positive clockwise from the x-axis, with the result that, for a diagonal to the left of a vertical, the cosine is negative. This reverses the signs of Equations 25 and 28. The axial force, P_d , is positive when tension, and negative when compression.

The general slope-deflection equations. The equations of equilibrium for any joint "n" may now be stated, making use of Figure 2.3 and the stiffness factors just derived. Since the forces and moments shown in Figure 2.3 are those applied by the joint, and since the stiffnesses have been derived on the same basis, it is convenient to sum and equate to zero these forces and moments, rather than those applied to the joint.

$$\Sigma M_x = 0$$

$$\begin{aligned} M_{n-1} \sin \theta_{n-1} - M'_n \sin \theta_n - T_{n-1} \cos \theta_{n-1} + T'_n \cos \theta_n + (nS'_{xx} + nS''_{xx})\alpha_n \\ + (nS'_{xy} + nS''_{xy})\beta_n + (nS'_{xz} + nS''_{xz})(\delta_n - \delta'_n) + nS_x = 0 \end{aligned} \quad 30$$

$$\Sigma M_y = 0$$

$$\begin{aligned} & -M_{n-1} \cos \theta_{n-1} + M'_n \cos \theta_n - T_{n-1} \sin \theta_{n-1} + T_n \sin \theta_n + (nS'_{xy} + nS''_{xy})\alpha_n \\ & + (nS'_{yy} + nS''_{yy})\beta_n + (nS'_{yz} + nS''_{yz})(\delta_n - \delta'_n) = 0 \end{aligned} \quad 31$$

$$\Sigma F_z = 0$$

$$\begin{aligned} & -\frac{(M'_{n-1} - M_{n-1})}{L_{n-1}} - \frac{P_{n-1}}{L_{n-1}}(\delta_n - \delta_{n-1}) + \frac{(M'_n - M_n)}{L_n} + \frac{P_n}{L_n}(\delta_{n+1} - \delta_n) \\ & + (nS'_{xz} + nS''_{xz})\alpha_n + (nS'_{yz} + nS''_{yz})\beta_n + (nS'_{zz} + nS''_{zz})(\delta_n - \delta'_n) \\ & + nS_z = 0 \end{aligned} \quad 32$$

The moments and torques in these equations, defined by Equations 9 through 14, are now expressed as follows, incorporating the relationships of Equations 15:

$$\begin{aligned} M_n &= (\delta_{n+1} - \delta_n) \frac{K'_n}{L_n} + K_n(\alpha_{n+1} \sin \theta_n - \beta_{n+1} \cos \theta_n) \\ &- K_n C_n(-\alpha_n \sin \theta_n + \beta_n \cos \theta_n) - (\delta'_{n+1} - \delta'_n) \frac{K'_n}{L_n} \end{aligned} \quad 33$$

$$\begin{aligned} M'_n &= -(\delta_{n+1} - \delta_n) \frac{K'_n}{L_n} - K_n C_n(\alpha_{n+1} \sin \theta_n - \beta_{n+1} \cos \theta_n) \\ &+ K_n(-\alpha_n \sin \theta_n + \beta_n \cos \theta_n) + (\delta'_{n+1} - \delta'_n) \frac{K'_n}{L_n} \end{aligned} \quad 34$$

$$\begin{aligned} M_{n-1} &= (\delta_n - \delta_{n-1}) \frac{K'_{n-1}}{L_{n-1}} + K_{n-1}(\alpha_n \sin \theta_{n-1} - \beta_n \cos \theta_{n-1}) \\ &- K_{n-1} C_{n-1}(-\alpha_{n-1} \sin \theta_{n-1} + \beta_{n-1} \cos \theta_{n-1}) - (\delta'_n - \delta'_{n-1}) \frac{K'_{n-1}}{L_{n-1}} \end{aligned} \quad 35$$

$$\begin{aligned}
M'_{n-1} = & -(\delta_n - \delta_{n-1}) \frac{K'_{n-1}}{L_{n-1}} - K_{n-1}C_{n-1}(\alpha_n \sin \theta_{n-1} - \beta_n \cos \theta_{n-1}) \\
& + K_{n-1}(-\alpha_{n-1} \sin \theta_{n-1} + \beta_{n-1} \cos \theta_{n-1}) + (\delta'_n - \delta'_{n-1}) \frac{K'_{n-1}}{L_{n-1}}
\end{aligned} \tag{36}$$

$$T_n = R_n(-\alpha_{n+1} \cos \theta_n - \beta_{n+1} \sin \theta_n + \alpha_n \cos \theta_n + \beta_n \sin \theta_n) \tag{37}$$

$$T_{n-1} = R_{n-1}(-\alpha_n \cos \theta_{n-1} - \beta_n \sin \theta_{n-1} + \alpha_{n-1} \cos \theta_{n-1} + \beta_{n-1} \sin \theta_{n-1}) \tag{38}$$

The equilibrium equations may now be stated in terms of stiffnesses and displacements:

$$\Sigma M_x = 0$$

$$\begin{aligned}
& (\delta_n - \delta_{n-1}) \frac{K'_{n-1}}{L_{n-1}} \sin \theta_{n-1} + K_{n-1} \sin \theta_{n-1} (\alpha_n \sin \theta_{n-1} - \beta_n \cos \theta_{n-1}) \\
& - K_{n-1} C_{n-1} \sin \theta_{n-1} (-\alpha_{n-1} \sin \theta_{n-1} + \beta_{n-1} \cos \theta_{n-1}) - (\delta'_n - \delta'_{n-1}) \frac{K'_{n-1}}{L_{n-1}} \sin \theta_{n-1} \\
& + (\delta_{n+1} - \delta_n) \frac{K'_n}{L_n} \sin \theta_n + K_n C_n \sin \theta_n (\alpha_{n+1} \sin \theta_n - \beta_{n+1} \cos \theta_n) \\
& - K_n \sin \theta_n (-\alpha_n \sin \theta_n + \beta_n \cos \theta_n) - (\delta'_{n+1} - \delta'_n) \frac{K'_n}{L_n} \sin \theta_n \\
& - R_{n-1} \cos \theta_{n-1} (-\alpha_n \cos \theta_{n-1} - \beta_n \sin \theta_{n-1} + \alpha_{n-1} \cos \theta_{n-1} + \beta_{n-1} \sin \theta_{n-1}) \\
& + R_n \cos \theta_n (-\alpha_{n+1} \cos \theta_n - \beta_{n+1} \sin \theta_n + \alpha_n \cos \theta_n + \beta_n \sin \theta_n) \\
& + (nS'_{xx} + nS''_{xx})\alpha_n + (nS'_{xy} + nS''_{xy})\beta_n + (nS'_{xz} + nS''_{xz})(\delta_n - \delta'_n) \\
& + nS_x = 0
\end{aligned}$$

Grouping according to displacements,

$$\begin{aligned}
& \delta_{n-1} \left[-\frac{K'_{n-1}}{L_{n-1}} \sin \theta_{n-1} \right] + \alpha_{n-1} [K_{n-1} C_{n-1} \sin^2 \theta_{n-1} - R_{n-1} \cos^2 \theta_{n-1}] \\
& + \beta_{n-1} [-(K_{n-1} C_{n-1} + R_{n-1}) \sin \theta_{n-1} \cos \theta_{n-1}] \\
& + \delta_n \left[\frac{K'_{n-1}}{L_{n-1}} \sin \theta_{n-1} - \frac{K'_n}{L_n} \sin \theta_n + nS'_{xz} + nS''_{xz} \right] \\
& + \alpha_n [K_{n-1} \sin^2 \theta_{n-1} + K_n \sin^2 \theta_n + R_{n-1} \cos^2 \theta_{n-1} + R_n \cos^2 \theta_n + nS'_{xx} + nS''_{xx}] \\
& + \beta_n [(-K_{n-1} + R_{n-1}) \sin \theta_{n-1} \cos \theta_{n-1} + (-K_n + R_n) \sin \theta_n \cos \theta_n + nS''_{xy}] \\
& + \delta_{n+1} \left[\frac{K'_n}{L_n} \sin \theta_n \right] + \alpha_{n+1} [K_n C_n \sin^2 \theta_n - R_n \cos^2 \theta_n] \\
& + \beta_{n+1} [-(K_n C_n + R_n) \sin \theta_n \cos \theta_n] = \left[(\delta'_n - \delta'_{n-1}) \frac{K'_{n-1}}{L_{n-1}} \sin \theta_{n-1} \right. \\
& \left. + (\delta'_{n+1} - \delta'_n) \frac{K'_n}{L_n} \sin \theta_n + \delta'_n (nS'_{xz} + nS''_{xz}) - nS_x \right]
\end{aligned} \tag{39}$$

Similarly,

$$\Sigma M_y = 0$$

$$\begin{aligned}
& \delta_{n-1} \left[\frac{K'_{n-1}}{L_{n-1}} \cos \theta_{n-1} \right] + \alpha_{n-1} [-(C_{n-1} K_{n-1} + R_{n-1}) \sin \theta_{n-1} \cos \theta_{n-1}] \\
& + \beta_{n-1} [C_{n-1} K_{n-1} \cos^2 \theta_{n-1} - R_{n-1} \sin^2 \theta_{n-1}] \\
& + \delta_n \left[-\frac{K'_{n-1}}{L_{n-1}} \cos \theta_{n-1} + \frac{K'_n}{L_n} \cos \theta_n + nS''_{yz} \right] \\
& + \alpha_n [(-K_{n-1} + R_{n-1}) \sin \theta_{n-1} \cos \theta_{n-1} + (-K_n + R_n) \sin \theta_n \cos \theta_n + nS''_{xy}] \\
& + \beta_n [K_{n-1} \cos^2 \theta_{n-1} + R_{n-1} \sin^2 \theta_{n-1} + K_n \cos^2 \theta_n + R_n \sin^2 \theta_n + nS'_{yy} + nS''_{yy}] \\
& + \delta_{n+1} \left[-\frac{K'_n}{L_n} \cos \theta_n \right] + \alpha_{n+1} [-(C_n K_n + R_n) \sin \theta_n \cos \theta_n] \\
& + \beta_{n+1} [C_n K_n \cos^2 \theta_n - R_n \sin^2 \theta_n] = \left[-(\delta'_n - \delta'_{n-1}) \frac{K'_{n-1}}{L_{n-1}} \cos \theta_{n-1} \right.
\end{aligned}$$

$$-(\delta'_{n+1} - \delta'_n) \frac{K'_n}{L_n} \cos \theta_n + \delta'_{n+1} S''_{yz} \quad 40$$

and

$$\Sigma F_z = 0$$

$$\begin{aligned} & \delta_{n-1} \left[\frac{P_{n-1}}{L_{n-1}} - \frac{2K'_{n-1}}{L^2_{n-1}} \right] + \alpha_{n-1} \left[\frac{K'_{n-1}}{L_{n-1}} \sin \theta_{n-1} \right] + \beta_{n-1} \left[-\frac{K'_{n-1}}{L_{n-1}} \cos \theta_{n-1} \right] \\ & + \delta_n \left[-\frac{P_{n-1}}{L_{n-1}} - \frac{P_n}{L_n} + \frac{2K'_{n-1}}{L^2_{n-1}} + \frac{2K'_n}{L^2_n} + nS'_{zz} + nS''_{zz} \right] \\ & + \alpha_n \left[\frac{K'_{n-1}}{L_{n-1}} \sin \theta_{n-1} - \frac{K'_n}{L_n} \sin \theta_n + nS'_{xz} + nS''_{xz} \right] \\ & + \beta_n \left[-\frac{K'_{n-1}}{L_{n-1}} \cos \theta_{n-1} + \frac{K'_n}{L_n} \cos \theta_n + nS''_{yz} \right] + \delta_{n+1} \left[\frac{P_n}{L_n} - \frac{2K'_n}{L^2_n} \right] \\ & + \alpha_{n+1} \left[-\frac{K'_n}{L_n} \sin \theta_n \right] + \beta_{n+1} \left[\frac{K'_n}{L_n} \cos \theta_n \right] \\ & = \left[2(\delta'_n - \delta'_{n-1}) \frac{K'_{n-1}}{L^2_{n-1}} - 2(\delta'_{n+1} - \delta'_n) \frac{K'_n}{L^2_n} + \delta'_n(nS'_{zz} + nS''_{zz}) - nS_z \right] \end{aligned} \quad 41$$

It may be observed that the unknowns and their coefficients have now been effectively separated. Note that although the axial forces P in the chord are compression, the derivation (see Figure 2.3) dictates that positive rather than negative values be used in the above equation.

Special equations for joints adjacent to end joints. Certain conditions exist at the end joints which must be evaluated and incorporated into the equilibrium equations for the adjacent joints.

First, the deflections of these joints are equal to zero; that is,

$$\delta_i = 0$$

$$\delta_{i+1} = 0$$

where "i" denotes the left, and "i + 1" the right, end joint of the truss, and where "i + 1" is numerically equal to the total number of joints in the truss, including the end joints.

Some assumption must be made regarding the degrees of fixity of the top chord at the end joints, both in bending and in torsion. The simplest such assumption is that the member is either fixed or free to rotate.

Consider bending: if the member is fixed, then its stiffness is that of a member with the far end fixed, represented previously by the symbol K . If the member is free to rotate at the end joint, then the stiffness is the modified stiffness with the far end hinged (K'''), and is derived as follows:

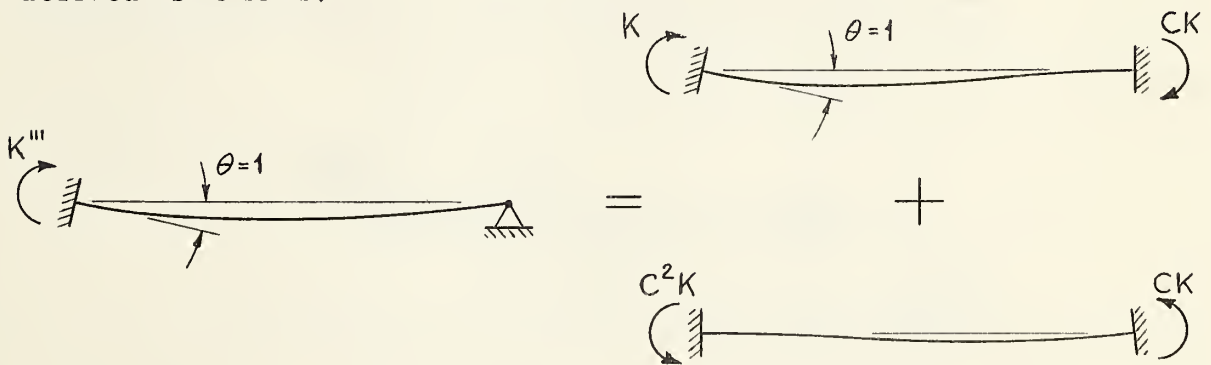


Figure 2.16

From Figure 2.16, $K''' = K(1 - C^2)$. It is apparent, then, that the

bending stiffness of the end member might be either K or $K(1 - C^2)$, depending upon which assumption is made. To facilitate modification of the general slope-deflection equations, it is possible to introduce an arbitrary constant "a" into the stiffness expression thus:

$$\text{bending stiffness} = K(1 - aC^2).$$

Obviously, when $a = 0$, the stiffness is simply K , and when $a = 1$, the stiffness is $K(1 - C^2)$. Accordingly, it may be stated that for a fixed end, $a = 0$, and for a hinged end $a = 1$. The above expression may now be incorporated into the general equations as the rotational stiffness of the end member.

Using the same constant "a", the stiffness factor K' for antisymmetrical bending may be modified according to the condition of fixity at the end joints. For a fixed end, this stiffness is equal to $K(1 + C)$. Figure 2.17 illustrates the case where the member is free to rotate at the end joint. It has been shown previously that

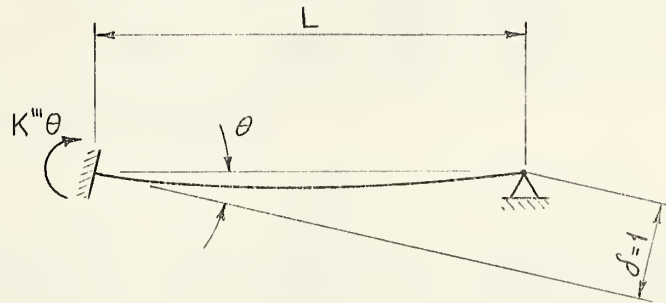


Figure 2.17

$K''' = K(1 - C^2)$. For small angles, $\theta = \frac{\delta}{L}$, and the bending moment

produced by a unit relative deflection is equal to $\frac{K}{L}(1 - C^2)$, or $\frac{K'}{L}(1 - C)$. Hence the stiffness factor relating end moments and relative lateral deflection of the ends of the member may be either $\frac{K'}{L}$ or $\frac{K'}{L}(1 - C)$, depending on the condition of fixity. The constant "a", then, can be incorporated as follows: $\frac{K'}{L}(1 - aC)$. Now if $a = 0$, the stiffness factor is $\frac{K'}{L}$, whereas if $a = 1$, the stiffness factor is $\frac{K'}{L}(1 - C)$.

The constant "a" may furthermore be used to modify the lateral force produced by a unit relative deflection of panel points, in order that Equation 41 may be conveniently applied to the end panels.

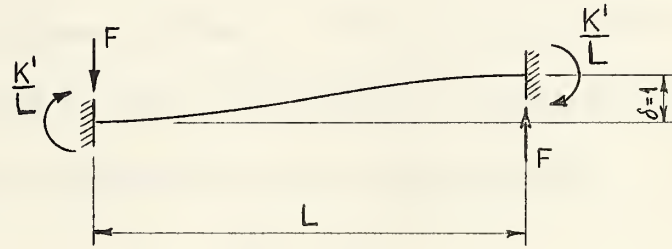


Figure 2.18

From Figure 2.18, it is obvious that the force "F" produced by a unit relative deflection as shown, is equal to $\frac{2K'}{L^2}$. Suppose, however, that one end of the member is hinged:

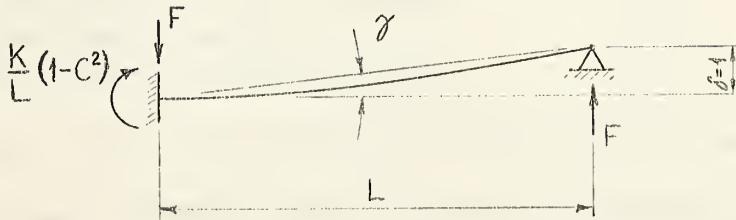


Figure 2.19

The moment produced at the fixed support is, for a small rotation

$\gamma = \frac{1}{L}$, equal to $\frac{K}{L}(1 - C^2)$. Employing the relationship that $K' = K(1 + C)$, then the force $F = \frac{K'}{L^2}(1 - C)$. Thus some expression containing "a" must become equal to $\frac{2K'}{L^2}$ when $a = 0$, and must become equal to $\frac{K'}{L^2}(1 - C)$ when $a = 1$. Such an expression could be as follows:

$$\frac{K'}{L^2}[2(1 - a) + a(1 - C)] = \frac{K'}{L^2}[2 - a(1 + C)]$$

A similar arbitrary constant may be used to specify the end condition in torsion. If the end is fixed, then the torsional stiffness is equal to R , as previously defined. If the end is free, then the torsional stiffness is equal to zero. Introducing "b" as the required constant, the torsional stiffness may be represented thus:

$$R(1 - b)$$

It is evident that when $b = 0$, the stiffness is R , and when $b = 1$, the stiffness is zero.

The use of the constants "a" and "b" may now be summarized as follows, with reference to the end joint:

Member fixed in bending	$a = 0$
Member free to rotate in bending	$a = 1$
Member fixed in torsion	$b = 0$
Member free to rotate in torsion	$b = 1$

Finally, it should be noted that the above discussion has referred to the degree of fixity between the member and the end joint, and that the deflection δ and the rotations α and β of that joint are equal to zero.

The modified equations for the first interior joints are as

follows: For joint 2:

$$\Sigma M_x = 0$$

$$\begin{aligned} & \delta_2 \left[\frac{K'_1}{L_1} (1 - aC_1) \sin \theta_1 - \frac{K'_2}{L_2} \sin \theta_2 + {}_2S'_{xz} + {}_2S''_{xz} \right] \\ & + \alpha_2 [K_1(1 - aC_1^2) \sin^2 \theta_1 + K_2 \sin^2 \theta_2 + R_1(1 - b) \cos^2 \theta_1 + R_2 \cos^2 \theta_2 \\ & + {}_2S'_{xx} + {}_2S''_{xx}] + \beta_2 [\{-K_1(1 - aC_1^2) + R_1(1 - b)\} \sin \theta_1 \cos \theta_1 \\ & + (-K_2 + R_2) \sin \theta_2 \cos \theta_2 + {}_2S''_{xy}] + \delta_3 \left[\frac{K'_2}{L_2} \sin \theta_2 \right] + \alpha_3 [K_2 C_2 \sin^2 \theta_2 \\ & - R_2 \cos^2 \theta_2] + \beta_3 [-(K_2 C_2 + R_2) \sin \theta_2 \cos \theta_2] = \left[\delta'_2 \frac{K'_1}{L_1} (1 - aC_1) \sin \theta_1 \right. \\ & \left. + (\delta'_3 - \delta'_2) \frac{K'_2}{L_2} \sin \theta_2 + \delta'_2 ({}_2S'_{xz} + {}_2S''_{xz}) - {}_2S_x \right] \end{aligned} \quad 42$$

$$\Sigma M_y = 0$$

$$\begin{aligned} & \delta_2 \left[-\frac{K'_1}{L_1} (1 - aC_1) \cos \theta_1 + \frac{K'_2}{L_2} \cos \theta_2 + {}_2S''_{yz} \right] \\ & + \alpha_2 [\{-K_1(1 - aC_1^2) + R_1(1 - b)\} \sin \theta_1 \cos \theta_1 + (-K_2 + R_2) \sin \theta_2 \cos \theta_2 \\ & + {}_2S''_{xy}] + \beta_2 [K_1(1 - aC_1^2) \cos^2 \theta_1 + R_1(1 - b) \sin^2 \theta_1 + K_2 \cos^2 \theta_2 \\ & + R_2 \sin^2 \theta_2 + {}_2S'_{yy} + {}_2S''_{yy}] + \delta_3 \left[-\frac{K'_2}{L_2} \cos \theta_2 \right] \\ & + \alpha_3 [-(K_2 C_2 + R_2) \sin \theta_2 \cos \theta_2] + \beta_3 [K_2 C_2 \cos^2 \theta_2 - R_2 \sin^2 \theta_2] \\ & = \left[-\delta'_2 \frac{K'_1}{L_1} (1 - aC_1) \cos \theta_1 - (\delta'_3 - \delta'_2) \frac{K'_2}{L_2} \cos \theta_2 + \delta'_2 {}_2S''_{yz} \right] \end{aligned} \quad 43$$

$$\Sigma F_z = 0$$

$$\delta_2 \left[-\frac{P_1}{L_1} - \frac{P_2}{L_2} + \{2 - a(1 + C_1)\} \frac{K'_1}{L_1^2} + \frac{2K'_2}{L_2^2} + {}_2S'_{zz} + {}_2S''_{zz} \right]$$

$$\begin{aligned}
& + \alpha_2 \left[\frac{K'_1}{L_1} (1 - aC_1) \sin \theta_1 - \frac{K'_2}{L_2} \sin \theta_2 + {}_2S'_{xz} + {}_2S''_{xz} \right] \\
& + \beta_2 \left[-\frac{K'_1}{L_1} (1 - aC_1) \cos \theta_1 + \frac{K'_2}{L_2} \cos \theta_2 + {}_2S''_{yz} \right] + \delta_3 \left[\frac{P_2}{L_2} - \frac{2K'_2}{L_2^2} \right] \\
& + \alpha_3 \left[-\frac{K'_2}{L_2} \sin \theta_2 \right] + \beta_3 \left[\frac{K'_2}{L_2} \cos \theta_2 \right] = \left[\delta'_2 \{2 - a(1 + C_1)\} \frac{K'_1}{L_1^2} \right. \\
& \left. - 2(\delta'_3 - \delta'_2) \frac{K'_2}{L_2^2} + \delta'_2 ({}_2S'_{zz} + {}_2S''_{zz}) - {}_2S_z \right]
\end{aligned} \tag{44}$$

For joint "i", the general equations become:

$$\begin{aligned}
\Sigma M_x &= 0 \\
\delta_{i-1} \left[-\frac{K'_{i-1}}{L_{i-1}} \sin \theta_{i-1} \right] &+ \alpha_{i-1} \left[K_{i-1} C_{i-1} \sin^2 \theta_{i-1} - R_{i-1} \cos^2 \theta_{i-1} \right] \\
&+ \beta_{i-1} \left[-(K_{i-1} C_{i-1} + R_{i-1}) \sin \theta_{i-1} \cos \theta_{i-1} \right] \\
&+ \delta_i \left[\frac{K'_{i-1}}{L_{i-1}} \sin \theta_{i-1} - \frac{K'_i}{L_i} (1 - aC_i) \sin \theta_i + iS'_{xz} + iS''_{xz} \right] \\
&+ \alpha_i \left[K_{i-1} \sin^2 \theta_{i-1} + K_i (1 - aC_i^2) \sin^2 \theta_i + R_{i-1} \cos^2 \theta_{i-1} + R_i (1 - b) \cos^2 \theta_i \right. \\
&+ iS'_{xx} + iS''_{xx} \left. \right] + \beta_i \left[(-K_{i-1} + R_{i-1}) \sin \theta_{i-1} \cos \theta_{i-1} \right. \\
&+ \left. \{-K_i (1 - aC_i^2) + R_i (1 - b)\} \sin \theta_i \cos \theta_i + iS''_{xy} \right] = \left[(\delta'_i - \delta'_{i-1}) \frac{K'_{i-1}}{L_{i-1}} \sin \theta_{i-1} \right. \\
&\left. - \delta'_i \frac{K'_i}{L_i} (1 - aC_i) \sin \theta_i + \delta'_i (iS'_{xz} + iS''_{xz}) - iS_x \right]
\end{aligned} \tag{45}$$

$$\begin{aligned}
\Sigma M_y &= 0 \\
\delta_{i-1} \left[\frac{K'_{i-1}}{L_{i-1}} \cos \theta_{i-1} \right] &+ \alpha_{i-1} \left[-(K_{i-1} C_{i-1} + R_{i-1}) \sin \theta_{i-1} \cos \theta_{i-1} \right] \\
&+ \beta_{i-1} \left[K_{i-1} C_{i-1} \cos^2 \theta_{i-1} - R_{i-1} \sin^2 \theta_{i-1} \right] \\
&+ \delta_i \left[-\frac{K'_{i-1}}{L_{i-1}} \cos \theta_{i-1} + \frac{K'_i}{L_i} (1 - aC_i) \cos \theta_i + iS''_{yz} \right] \\
&+ \alpha_i \left[(-K_{i-1} + R_{i-1}) \sin \theta_{i-1} \cos \theta_{i-1} + \{-K_i (1 - aC_i^2) + R_i (1 - b)\} \sin \theta_i \cos \theta_i \right. \\
&+ iS''_{xy} \left. \right] + \beta_i \left[K_{i-1} \cos^2 \theta_{i-1} + R_{i-1} \sin^2 \theta_{i-1} + K_i (1 - aC_i^2) \cos^2 \theta_i \right. \\
&+ \left. R_i (1 - b) \sin^2 \theta_i + iS'_{yy} + iS''_{yy} \right] = \left[-(\delta'_i - \delta'_{i-1}) \frac{K'_{i-1}}{L_{i-1}} \cos \theta_{i-1} \right.
\end{aligned}$$

$$+ \delta'_i (1 - aC_i) \frac{K'_i}{L_i} \cos \theta_i + \delta'_i i S''_{yz} \Big] \quad 46$$

$$\Sigma F_z = 0$$

$$\begin{aligned} & \delta_{i-1} \left[\frac{P_{i-1}}{L_{i-1}} - \frac{2K'_{i-1}}{L^2_{i-1}} \right] + \alpha_{i-1} \left[\frac{K'_{i-1}}{L_{i-1}} \sin \theta_{i-1} \right] + \beta_{i-1} \left[-\frac{K'_{i-1}}{L_{i-1}} \cos \theta_{i-1} \right] \\ & + \delta_i \left[-\frac{P_{i-1}}{L_{i-1}} - \frac{P_i}{L_i} + \frac{2K'_{i-1}}{L^2_{i-1}} + \{2 - a(1 + C_i)\} \frac{K'_i}{L^2_i} + iS'_{zz} + iS''_{zz} \right] \\ & + \alpha_i \left[\frac{K'_{i-1}}{L_{i-1}} \sin \theta_{i-1} - (1 - aC_i) \frac{K'_i}{L_i} \sin \theta_i + iS'_{xz} + iS''_{xz} \right] \\ & + \beta_i \left[-\frac{K'_{i-1}}{L_{i-1}} \cos \theta_{i-1} + \frac{K'_i}{L_i} (1 - aC_i) \cos \theta_i + iS''_{yz} \right] \\ & = \left[2(\delta'_i - \delta'_{i-1}) \frac{K'_{i-1}}{L^2_{i-1}} + \delta'_i \{2 - a(1 + C_i)\} \frac{K'_i}{L^2_i} + \delta'_i (iS'_{zz} + iS''_{zz}) - iS_z \right] \end{aligned}$$

III. THE SYSTEM OF EQUATIONS

It has been demonstrated that for each interior joint of the truss, there are three equations of equilibrium. Boundary conditions have been incorporated into Equations 42 through 47 for the joints adjacent to the end joints. The resulting system of equations may be solved simultaneously for the three displacements of each joint.

IV. COLLAPSE LOAD OF THE BRIDGE

When the phenomenon of buckling occurs, the displacements of the top chord joints become very large. At this time, the determinant of the coefficients in the system of equations approaches zero. This

characteristic therefore permits an evaluation of the elastic buckling load. The procedure is as follows: some arbitrary load is applied to the bridge, and the determinant of the coefficients of the slope-deflection equations is computed. This is repeated with different loads until the load for which the determinant equals zero has been bracketed. Interpolation then gives the buckling load.

The buckling load thus found is independent of the initial deflections of the top chord joints, and of the bending moments in the floor beams. It may be thought of as the upper limit of the actual collapse load, for it is necessary that the buckling mode be perfectly symmetrical or antisymmetrical, and that no members of the truss have yielded. Yielding of any member, save the floor beams, causes a redistribution of stresses among adjacent members, and radically changes the stiffness of the member itself. Since actual structures probably never attain the ideal, some more accurate means must be found of estimating the true collapse load, or at least a lower limit.

To this end, the displacements computed by means of this analysis may be substituted into equations developed earlier, and the secondary stresses evaluated. The bending and torsional moments in the top chord may be determined from Equations 9 through 14. The moments in the verticals are found by superimposing the effects of the bending moment in the floor beam (Equations 17 and 18), and of the cross-frame stiffnesses (Equations 20 through 23).

An IBM 1620 electronic digital computer program for the solution of the displacements is found in Appendix A.

CHAPTER III

THE EXPERIMENTAL PROGRAM

An experimental program was undertaken in order to determine the ultimate capacity of a typical pony-truss as well as to provide information concerning its general behavior. This testing program was a sequel to that carried out in 1959 by Mr. F. Lukawitski. The procedure devised by Mr. Lukawitski during the 1959 test was followed for the two 1960 tests, although a reappraisal of his data and results indicated a need for certain changes in the instrumentation.

The 1959, July 1960 and October 1960 tests are referred to hereafter as the 1959, July and October tests, respectively.

I. THE TEST BRIDGE

The bridge tested was the one used previously by Mr. Lukawitski. It was a Government of Alberta Standard 80-ft. Light Highway Span - 16 ft. Roadway (Figure 3.1), typical of many such structures currently in use in the Province of Alberta. Prior to its removal to the test site at Lambton Yard, Edmonton, the bridge was in service on a secondary road in Central Alberta, where it had at some time been damaged, resulting in relatively large initial lateral deflections of the upper chord of the south truss (Figure 3.2 - 1959 test). The 1959 test was carried out without repair of the truss, in

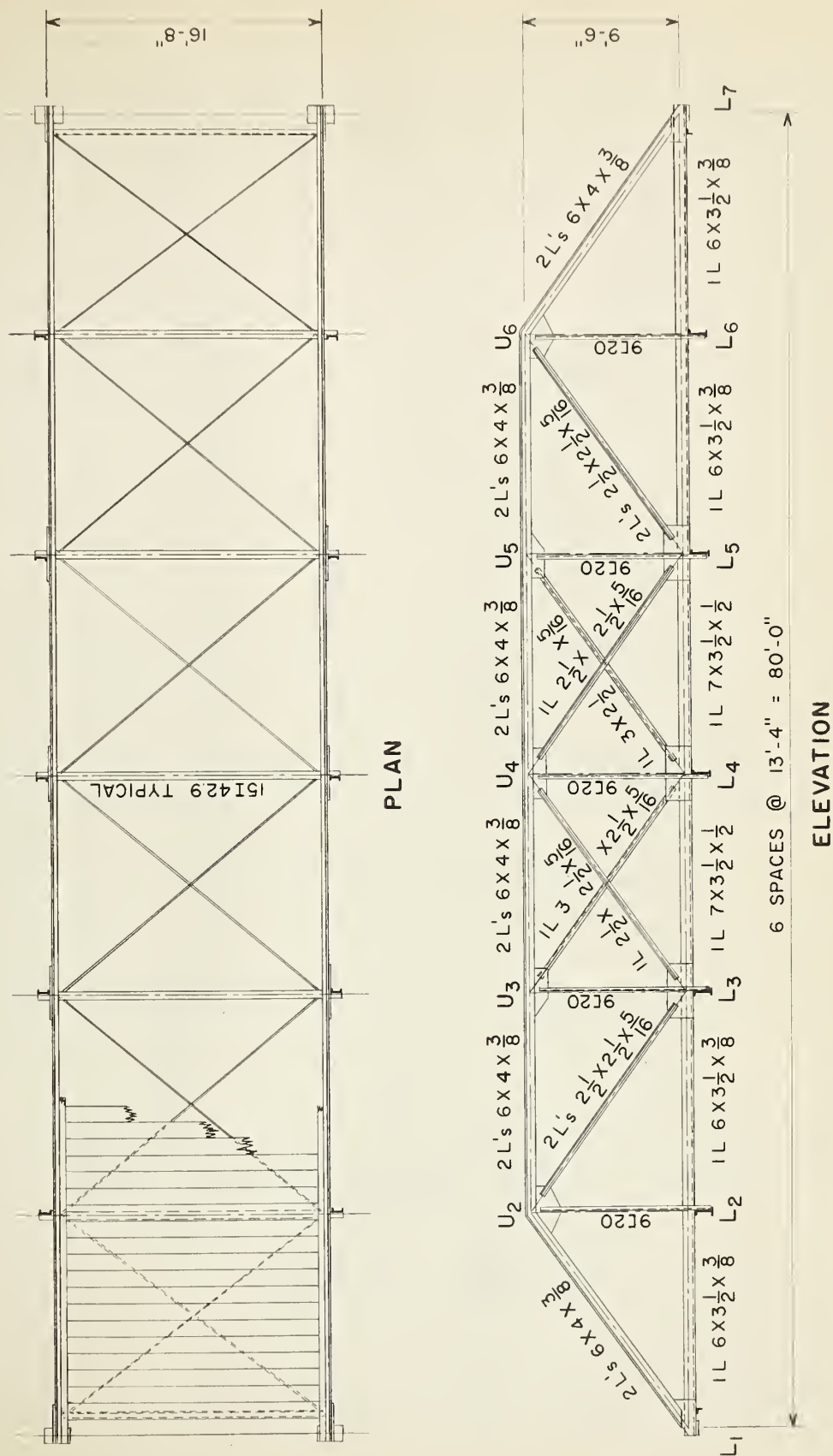


FIGURE 3.1 - DESIGN DRAWING OF TEST BRIDGE

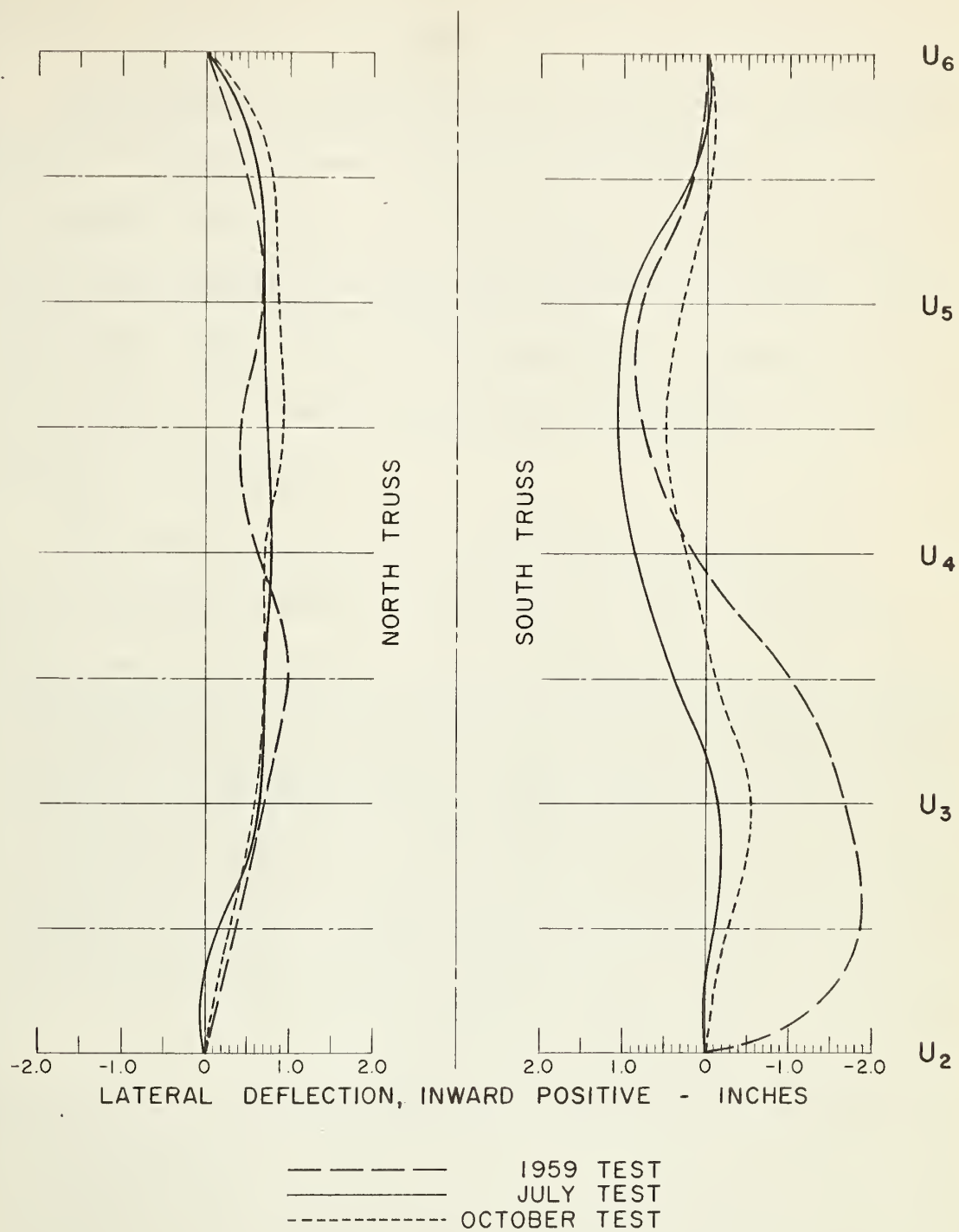


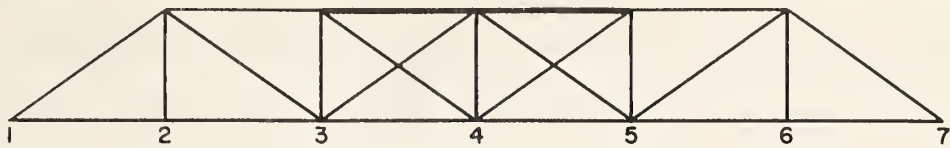
FIGURE 3.2 - TOP CHORD INITIAL LATERAL DEFLECTIONS

order that the effect of initial deflection or "crookedness" might be assessed.

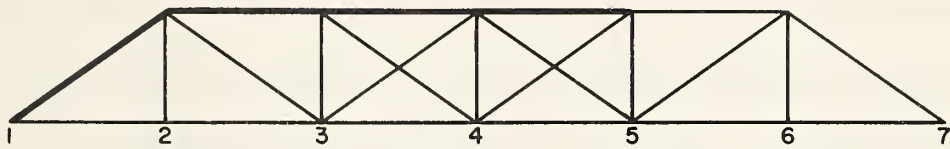
The July test. In preparation for the July test, it was deemed advisable to replace not only the severely buckled south chord, but also a portion of the north chord. As the upper chords were originally spliced at panel points U_3 and U_5 , the sections replaced were as shown in Figure 3.3. New pieces were fabricated for the purpose and the old ones discarded. The resulting upper chord configurations are shown in Figure 3.2 - July test.

The center floor beam had been loaded beyond its elastic capacity and had experienced considerable permanent deformation (Figure 3.4). The consequent rotational displacements of the ends of the beam tended to tilt the connected verticals toward the center-line of the truss, causing some difficulty in the replacement of the top chord, as well as adding to its initial deflection. A certain degree of correction was effected by pulling out the tops of the verticals by means of a power winch. Despite these problems, however, it was decided not to replace the floor beam, since it had presumably strain-hardened and could be expected to behave elastically up to at least the previous maximum load.

The October test. Repair of the bridge was again found necessary following the July test and before proceeding with the



NORTH TRUSS, JULY TEST
SOUTH TRUSS, OCTOBER TEST



SOUTH TRUSS, JULY TEST

REPLACED MEMBERS INDICATED BY HEAVY LINES

FIGURE 3.3 - TOP CHORD REPAIRS



FIGURE 3.4 - CENTER FLOOR BEAM BEFORE JULY TEST

October test. This time, though, only the south truss was considered so badly damaged as to warrant repair. Instead of using a new piece, the discarded section of the original north chord was straightened satisfactorily and reused in the south truss (Figure 3.3).

In view of the proposed load distribution for the October test, and certain questions arising from the July test, it appeared advisable to replace the center floor beam. This was accomplished by jacking and blocking of the adjacent floor beams and stringers. A substitute used beam, in good condition and identical in size and fabrication to the original, was found stockpiled nearby and was riveted in position with a minimum of difficulty.

As is evident from Figure 3.2, for none of the tests were there initially straight upper chords. Straightness of structural members appears to be, in a practical sense, a matter of degree, but even on this basis the south chord must be considered crooked for the 1959 test, and perhaps for even the July test. From observations of the erection procedures used in repair of the bridge, it was felt that no better approximations to straightness were attainable in practice, and that the initial upper chord configurations of the July and October tests probably were typical.

II. PREPARATION OF THE BRIDGE - INSTRUMENTATION

Following the pattern of the 1959 test, strain measurements on

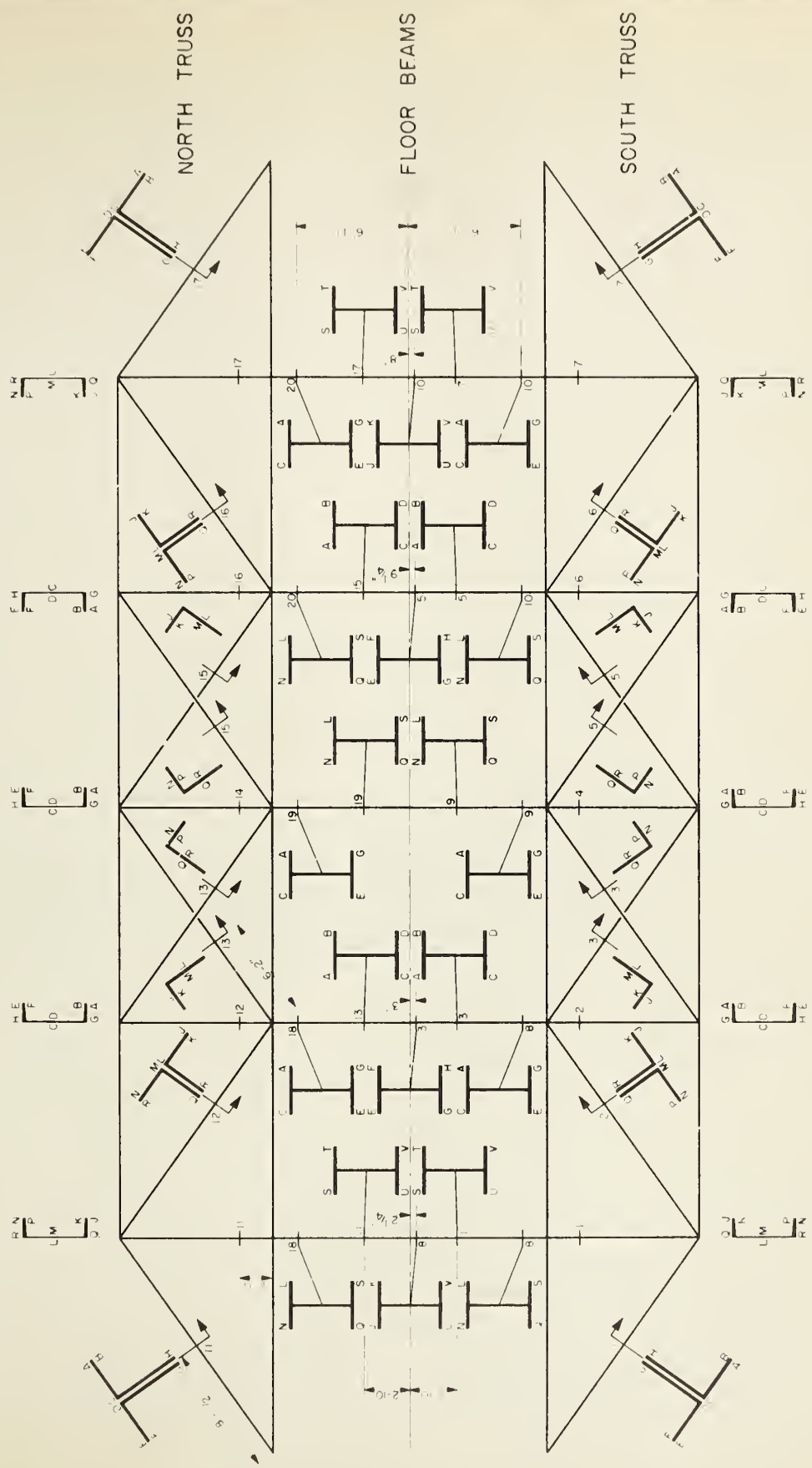
various members were taken by means of Baldwin SR-4 strain gauges. Additional gauges were applied at several locations, and many were disconnected because it was thought that little if any useful information could be derived from them. Figures 3.5 - 3.7 show the locations of the strain gauges.

Deflection measurements were taken by means of both dial indicators and scales.

Upper chords. Strain readings on the upper chords were not taken during either the July or the October test. Mr. Lukawitski had stated that the forces in these members were close to the theoretical values for a pin-jointed truss, and although there was some doubt as to the validity of his method of calculating the axial strains, this postulate was accepted. Further, there was no intention to compute the buckling load using Shanley's method, as illustrated by Mr. Lukawitski.

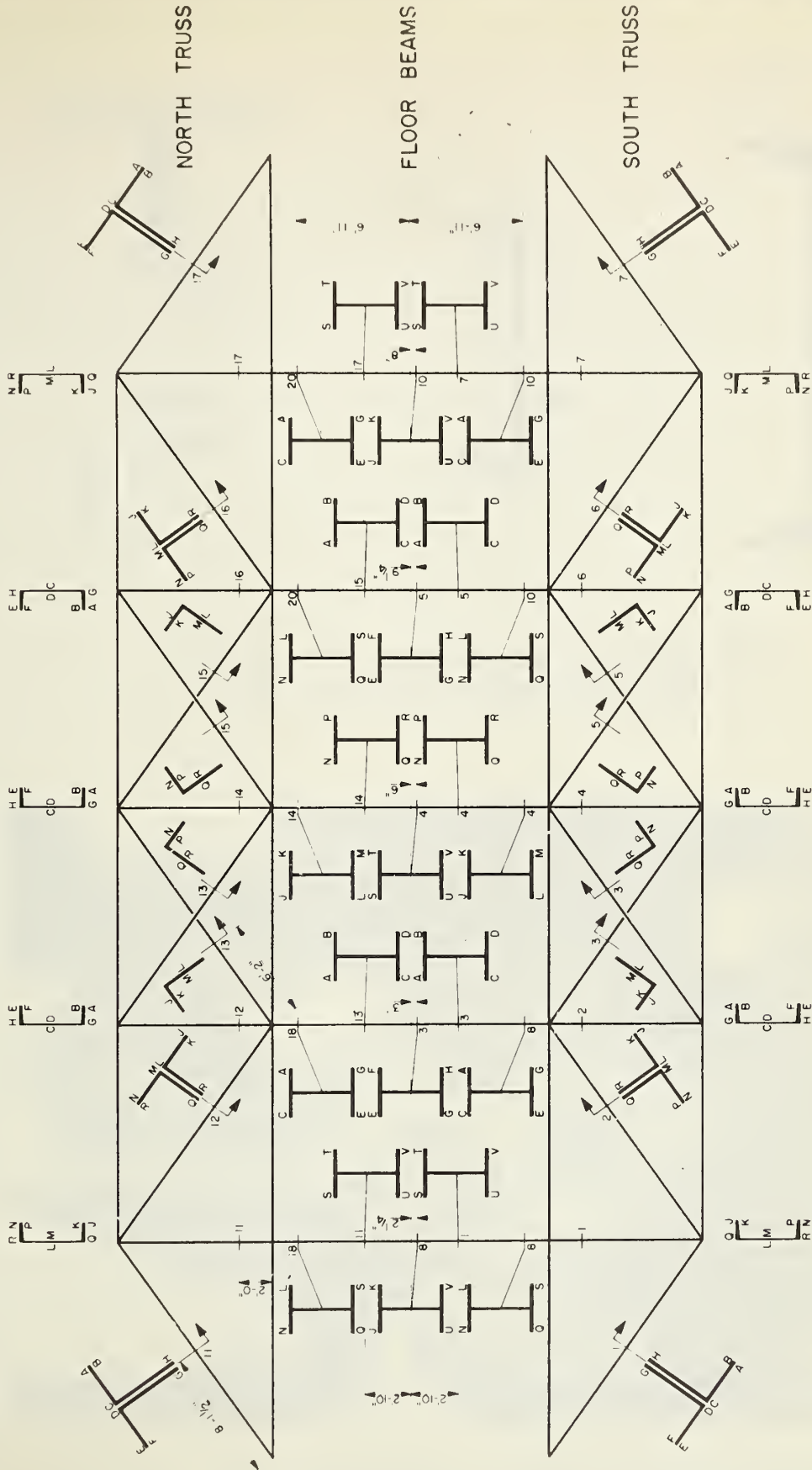
Measurements of lateral deflections, however, were carried out throughout the tests in the same manner as during the 1959 test. As shown in Figure 3.8, dial readings were supplemented by scale readings for the purpose of verification. Only the dial readings were intended for use in analysis.

Vertical members. The action of the vertical members was such that it became important to know the bending strains fairly accurately. Since single channels were used for these members, the



NUMERALS DESIGNATE STRAIN GAUGE STATIONS
LETTERS DESIGNATE INDIVIDUAL STRAIN GAUGES

FIGURE 3.5 - STRAIN GAUGE LOCATIONS, JULY TEST



NUMERALS DESIGNATE STRAIN GAUGE STATIONS
LETTERS DESIGNATE INDIVIDUAL STRAIN GAUGES

FIGURE 3.6 - STRAIN GAUGE LOCATIONS, OCTOBER TEST

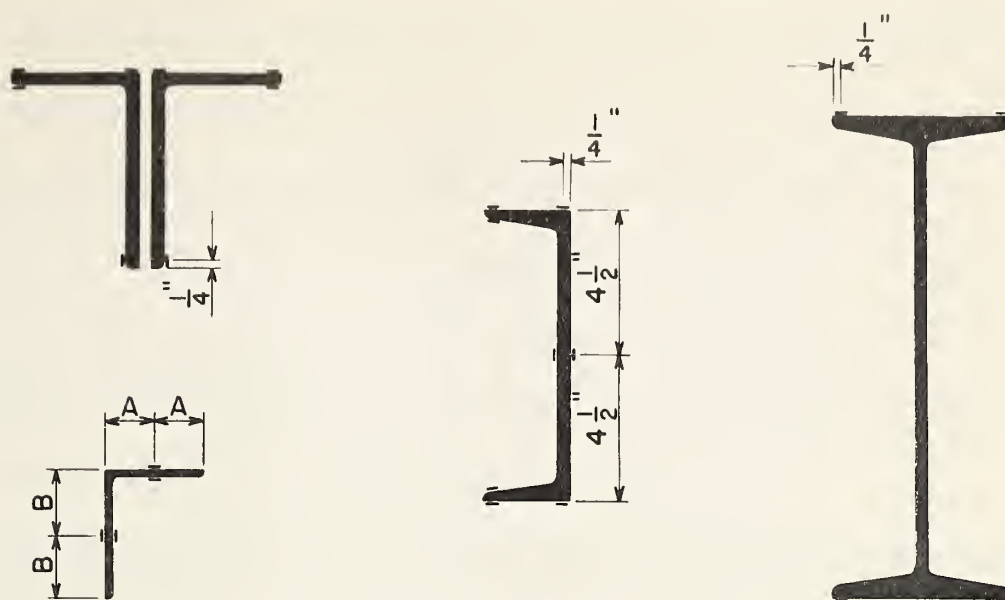


FIGURE 3.7 - STRAIN-GAUGE POSITIONS
ON TYPICAL CROSS-SECTIONS

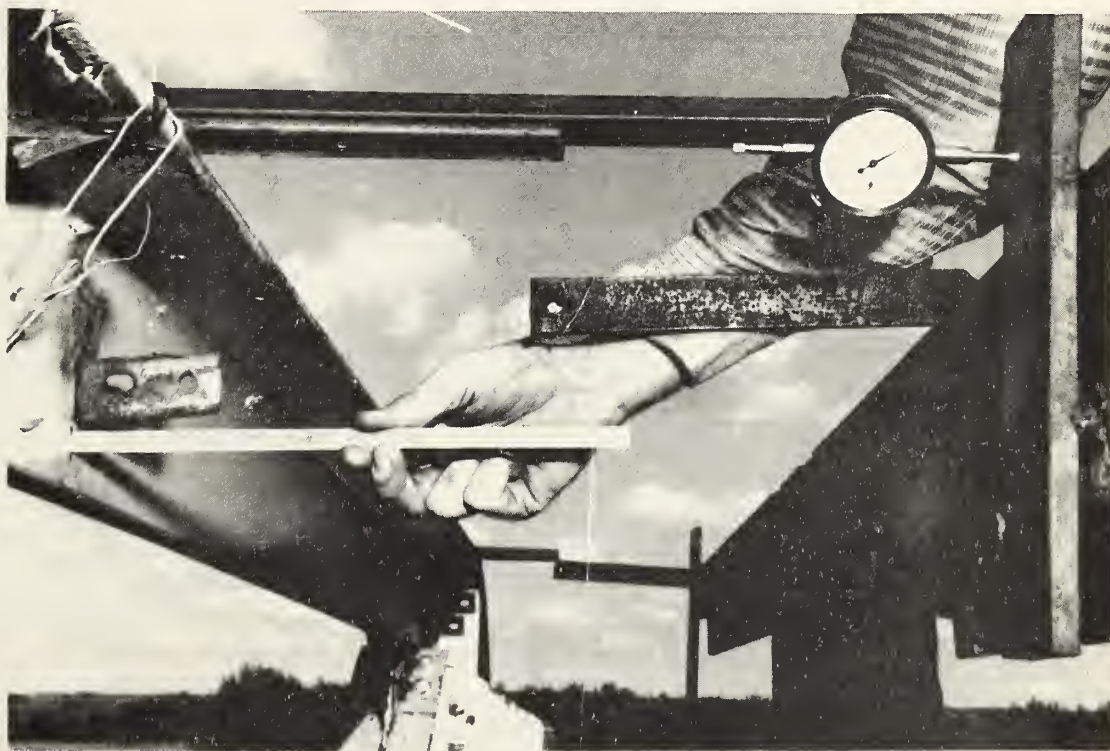


FIGURE 3.8 - MEASUREMENT OF TOP CHORD DEFLECTIONS

strains measured by Mr. Lukawitski were complicated by the presence of torsional components. It was, in fact, impossible to compute accurate bending strains using only the gauges applied prior to the 1959 test. Analysis showed (see Appendix B) that the effects of torsion, axial load, and bending about the weak axis could be virtually eliminated in the computations if additional strain readings were taken on the flanges at the heels of the channel.

At the time of the tests it also seemed desirable to know what proportions of the deflection of each upper chord panel point (and hence the top of each vertical) were due to bending of the vertical, rotation of the end of the floor beam and slippage of the joint, respectively. The instrumentation used in the 1959 test was incapable of supplying reliable data for this purpose. Other than the lateral deflections of the top chord, only the slip of the joint could be measured, and that inadequately. Laboratory tests suggested that deflection measurements taken as shown in Figure 3.9, in conjunction with data concerning the floor beams, could provide the required information. Subsequent study of the slope-deflection analysis, however, obviated the use of this information.

Floor beams. For the 1959 test, none of the floor beams had been adequately instrumented. One strain gauge station was located at each end of each beam. The center floor beam had two additional

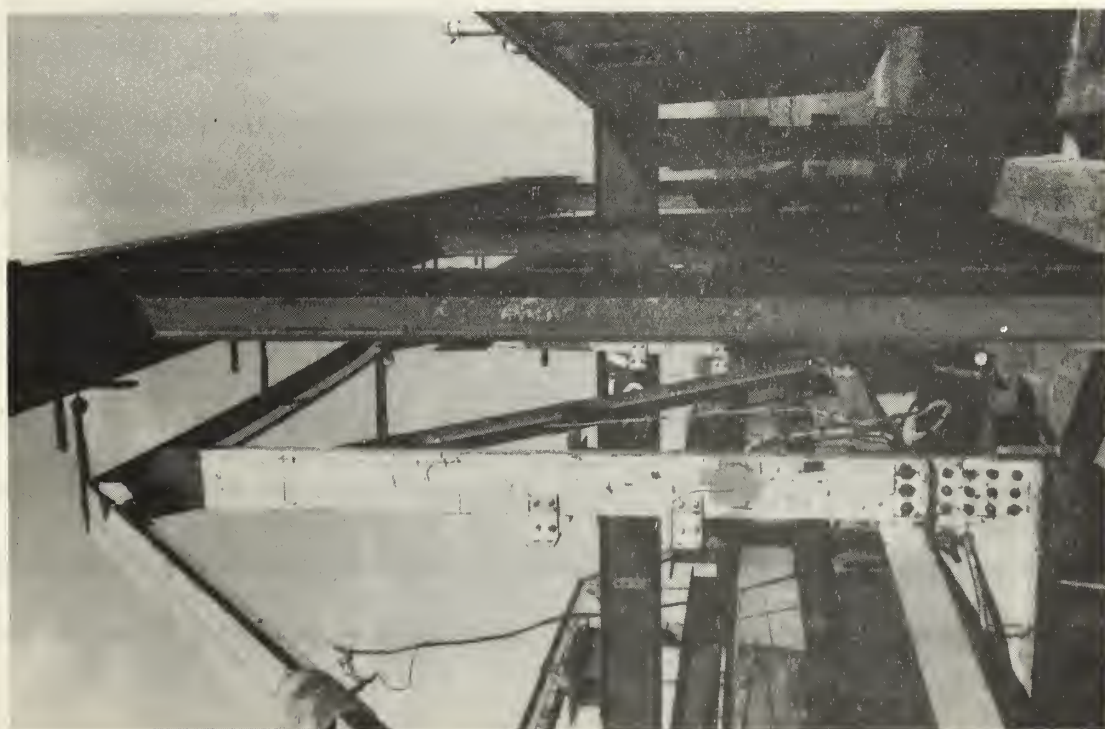


FIGURE 3.9 - MEASUREMENT OF
LATERAL DEFLECTIONS OF VERTICALS

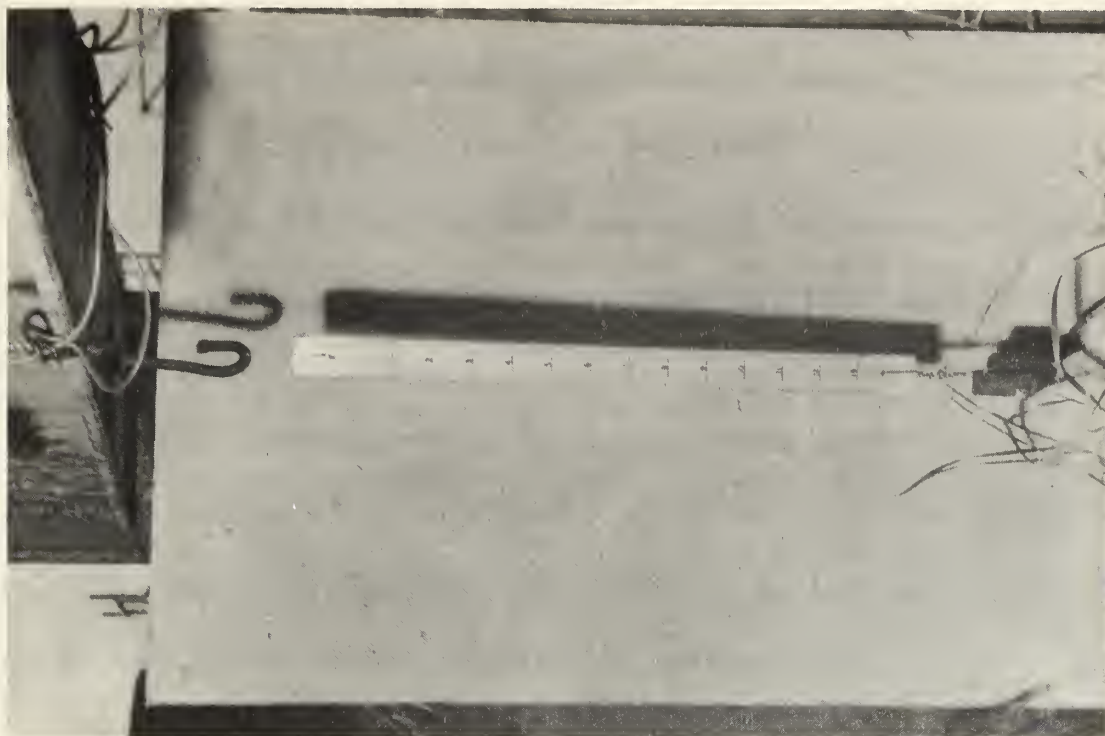


FIGURE 3.10 - MEASUREMENT OF
VERTICAL DEFLECTIONS

stations, located approximately at the two load points. To permit investigation of the possibility of load transfer to adjacent beams, more complete information was required concerning the bending moments in the beams. Moreover, the analysis in Appendix B being valid for wide-flange and I-sections, stations of only four gauges each were necessary (Figure 3.7). Three stations were added to each floor beam other than the center one, as shown in Figure 3.5. Between the July and October tests, an unfortunate omission was rectified with the addition of a gauge station near the middle of the center floor beam.

Vertical deflections were measured as in the 1959 test, by means of scales hanging from the ends and middle of each floor beam (Figure 3.10).

Other members. All of the web members, including the end diagonals, were important in the consideration of load transfer. Thus strain measurements were carried out for all of these members. Laboratory tests were carried out on both a single-angle and a double-angle member. The single-angle test verified the results of a similar test carried out by Mr. Lukawitski, but the double-angle test indicated that simply averaging the eight strain readings does not give a satisfactory indication of the axial load. This latter test notwithstanding, the instrumentation on the double-angle members was not altered, as no more suitable arrangement of gauges was discovered.

III. METHOD OF LOADING

In both 1960 tests, two fifty-ton hydraulic jacks, fitted with calibrated pressure gauges, were used to apply load to the bridge. The jacks were calibrated in the 200,000-pound Baldwin testing machine at the University of Alberta. As in the earlier test, the jacks were placed symmetrically six feet apart and reacted against the loading beams installed at that time. The entire reaction system is shown in Figure 3.11.

The July test. The loading used in the July test was identical to that used in the 1959 test; that is, the entire load was applied to the center floor beam. The photograph of Figure 3.12 shows the jacks in use.

The October test. For the October test, a different distribution of load was used. Several simple beams were arranged such that about 41.4% of the load was applied to the center floor beam, and about 29.3% to each of the two adjacent floor beams. The resulting loads on the five floor beams were in the ratio of 0 : 0.707 : 1.00 : 0.707 : 0, a sine-wave distribution. The erection drawing for this loading system is seen in Figure 3.13, while Figures 3.14 and 3.15 are photographs of it.

IV. THE TEST PROCEDURE

As in the 1959 test, the loading was carried out at night because



FIGURE 3.11 - REACTION SYSTEM



FIGURE 3.12 - JACKING OPERATION, JULY TEST

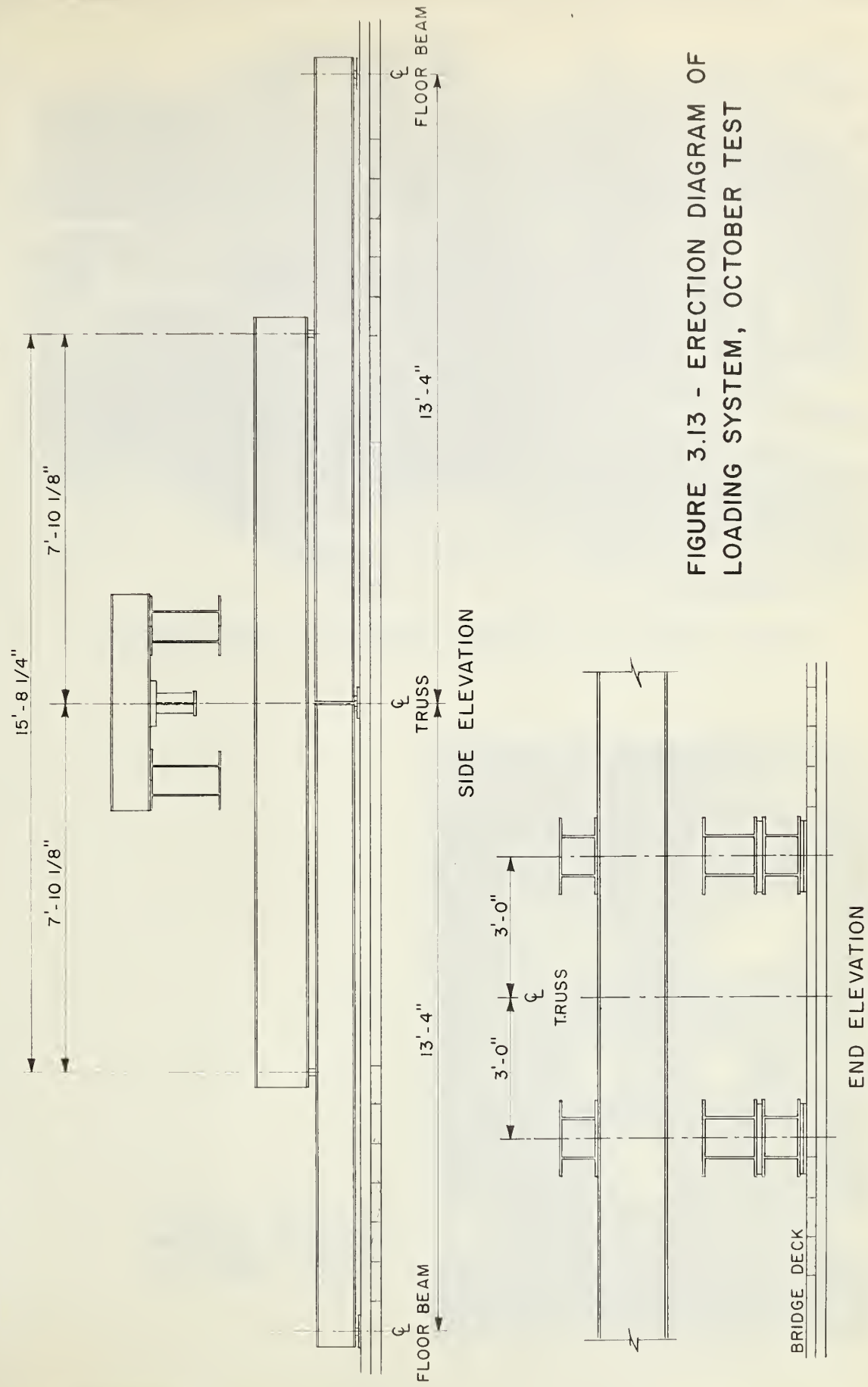


FIGURE 3.13 - ERECTION DIAGRAM OF
LOADING SYSTEM, OCTOBER TEST

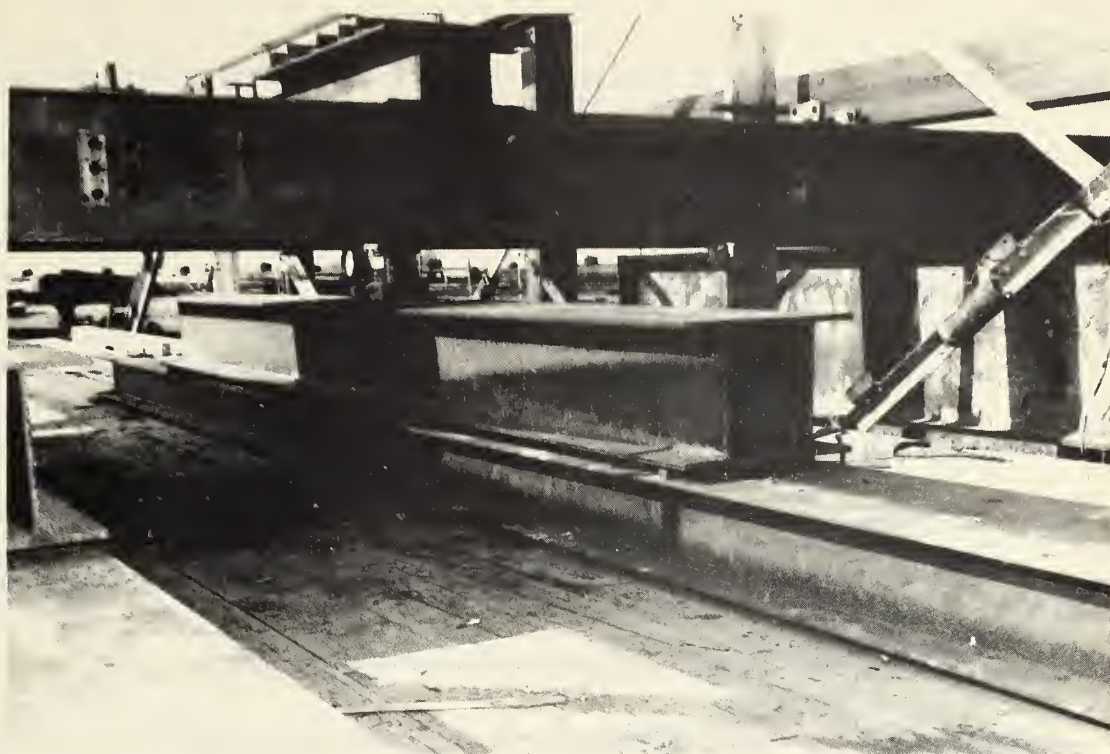


FIGURE 3.14 - LOADING SYSTEM, OCTOBER TEST

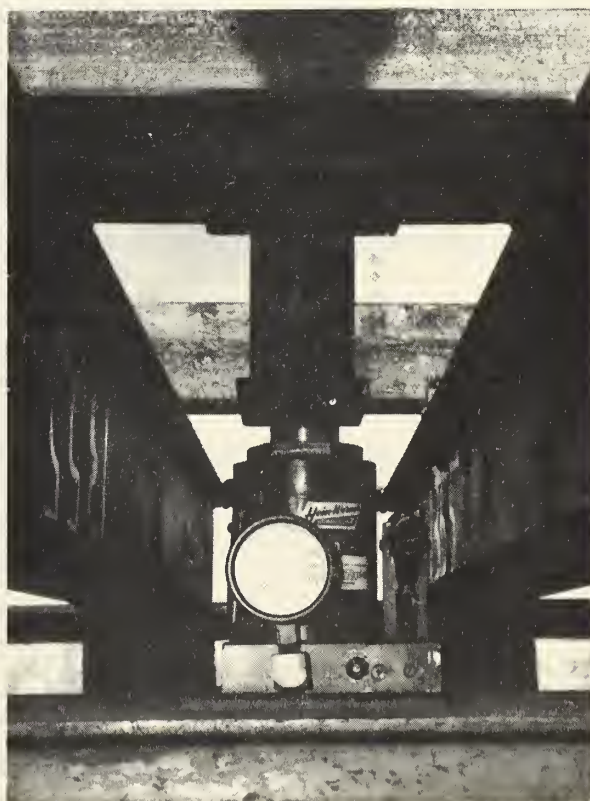


FIGURE 3.15 -
JACKING SYSTEM,
OCTOBER TEST

of the adverse effect of sunlight on the strain gauges. The night before each test, a trial loading was applied, and complete sets of readings taken, including zero readings before and after each loading. This procedure served as a check on the instruments and the gauges, as well as providing a correlation for actual test readings whenever they were in doubt during analysis of the data. In addition, on the evening of the test, the bridge was loaded conservatively and unloaded several times prior to the test itself.

Before applying the first load increment, initial readings were taken of all instrumentation. Load was then applied in five or ten-ton increments (see Chapter IV), complete sets of readings being taken for each increment.

Two persons operated the jacks, as shown in Figure 3.12. Each read his gauge aloud during increase of load in order that the rates of application would be approximately equal.

As the failure load was approached, strain and deflection readings could not be held constant, so recording of them was abandoned. Jacking was continued until the pressure gauges would not register any increase in load despite movement of the piston, and until buckling occurred.

CHAPTER IV

RESULTS OF LOAD TESTS

The results presented herein have been derived from the observations carried out during the tests described in the preceding chapter. It should be understood that, due to circumstances, the load tests were completed before a detailed study could be made of the slope-deflection analysis presented in Chapter II. Obviously, then, the test program could not be designed specifically to verify the theory; indeed, it has proven impossible to exactly correlate the experimental and analytical results, in those cases where any comparison at all was feasible.

Nevertheless, information obtained, but which does not pertain directly to the analysis, is presented in the interests of completeness, and with a view to exhibiting the general behavior under load of a single-span pony-truss bridge.

I. VERTICAL DEFLECTIONS

As stated in Chapter III, the vertical deflections of the lower chord panel points, and of the floor beam centerlines, were measured. Certain of these were recorded graphically during testing, in order that any indication of incipient collapse might be readily observed, as manifest by an abnormally rapid increase in deflection versus load.

The deflections have been tabulated in Tables 4.1 and 4.2, and the load-deflection relationships are illustrated diagrammatically in Figures 4.1 through 4.7.

A few words should be included concerning the accuracy of the vertical deflection readings. A comparison of the results of the July and October tests would suggest that somewhat less care was taken during the latter. The reader is referred particularly to Table 4.2 and to Figures 4.2, 4.3 and 4.5. Examining Table 4.2 more closely, it will be noted that the deflections recorded for the north truss exceed those recorded for the center of the floor beams. This is obviously impossible under the loading conditions imposed. When the deflections of the north truss are shown graphically, however, as in Figure 4.3, it becomes apparent that the initial readings were in error. Straight lines have been projected back to the zero axis, giving 0.5, 1.0 and 1.5 inches for three initial readings. It is impossible to more than merely speculate as to the exact causes of these errors; they may have been human or mechanical. The curves shown in Figure 4.4 and corresponding to those shown in Figure 4.3, then, have been corrected to zero as indicated by Figure 4.3.

One further comment is necessary with regard to accuracy. The curves for the north truss, July test, suffer a distinct and mutually similar discontinuity. Such a discontinuity might be produced by a sudden shift of load from that truss. If this were the case, the

TABLE 4.1
VERTICAL DEFLECTIONS, JULY TEST

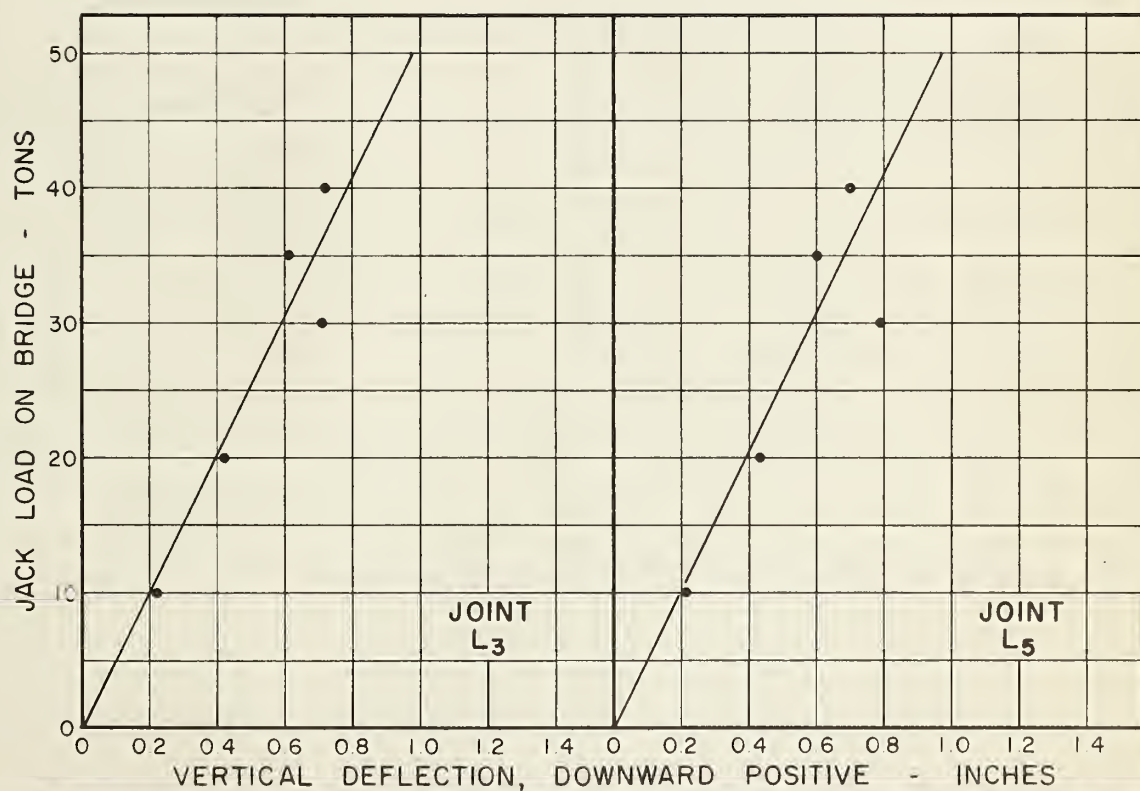
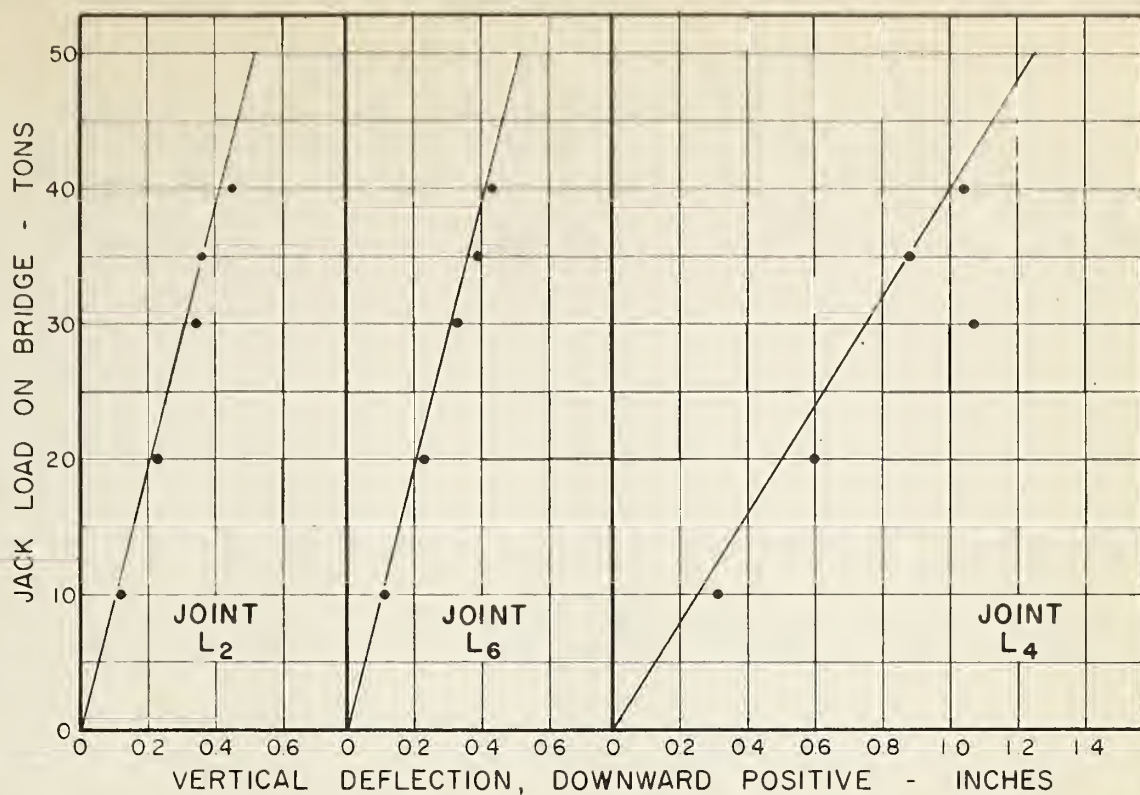
PANEL POINT	LOAD, TONS				
	10	20	30	35	40
NORTH TRUSS					
2	0.12	0.23	0.34	0.36	0.45
3	0.22	0.42	0.71	0.61	0.72
4	0.31	0.60	1.07	0.88	1.04
5	0.21	0.43	0.79	0.60	0.70
6	0.11	0.23	0.33	0.39	0.43
CENTER OF BEAM					
2	0.10	0.19	0.30	0.37	0.40
3	0.24	0.45	0.66	0.77	0.87
4	0.45	0.91	1.32	1.56	1.81
5	0.24	0.47	0.68	0.79	0.89
6	0.12	0.22	0.33	0.36	0.43
SOUTH TRUSS					
2	0.11	0.21	0.31	0.40	0.44
3	0.20	0.42	0.62	0.78	0.88
4	0.27	0.60	0.85	1.03	1.19
5	0.21	0.42	0.63	0.74	0.87
6	0.12	0.23	0.33	0.41	0.45

Deflections in inches, positive downward.

TABLE 4.2
VERTICAL DEFLECTIONS, OCTOBER TEST

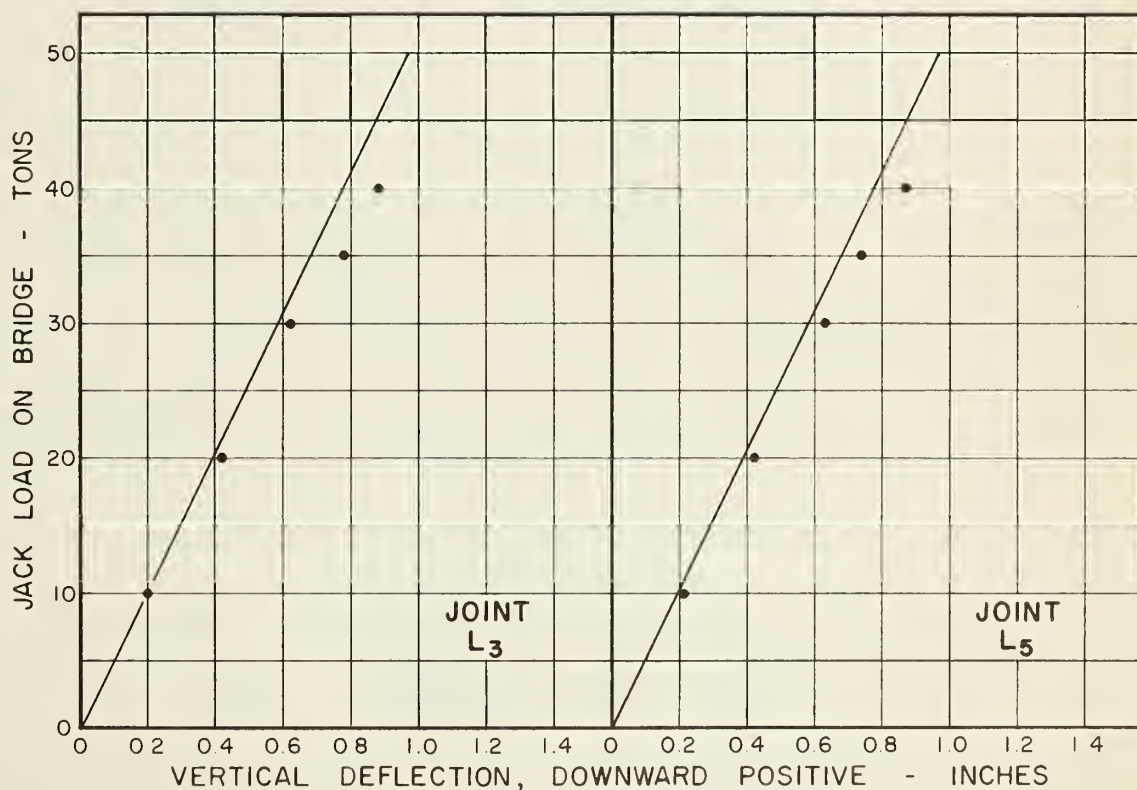
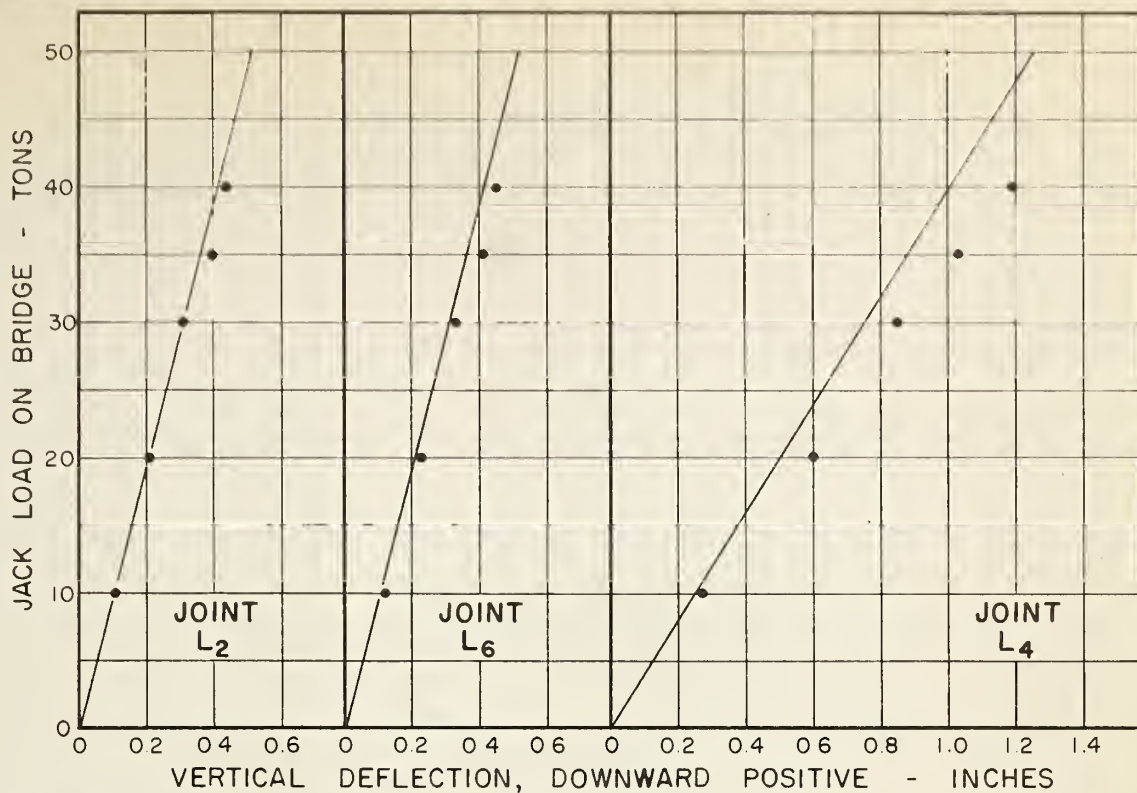
PANEL POINT	LOAD, TONS				
	10	20	30	40	50
NORTH TRUSS					
2	0.05	0.34	0.45	0.53	0.62
3	0.78	1.83	2.00	2.16	2.30
4	1.19	1.42	1.70	1.87	2.07
5	0.69	0.90	1.08	1.27	1.50
6	0.15	0.25	0.35	0.46	0.57
CENTER OF BEAM					
2	0.01	0.20	0.31	0.40	0.56
3	0.05	0.45	0.74	0.87	1.24
4	0.14	0.59	0.90	1.22	1.60
5	0.25	0.48	0.71	0.99	1.29
6	0.10	0.20	0.30	0.40	0.50
SOUTH TRUSS					
2	0.10	0.19	0.29	0.40	0.42
3	0.04	0.29	0.43	0.62	0.97
4	0.20	0.60	0.65	0.95	1.08
5	0.18	0.55	0.65	0.89	1.01
6	0.07	0.17	0.27	0.40	0.50

Deflections in inches, positive downward.



THEORETICAL DEFLECTIONS ILLUSTRATED BY SOLID LINES

FIGURE 4.1 - VERTICAL DEFLECTIONS OF NORTH TRUSS
JULY TEST



THEORETICAL DEFLECTIONS ILLUSTRATED BY SOLID LINES

FIGURE 4.2 - VERTICAL DEFLECTIONS OF SOUTH TRUSS
JULY TEST

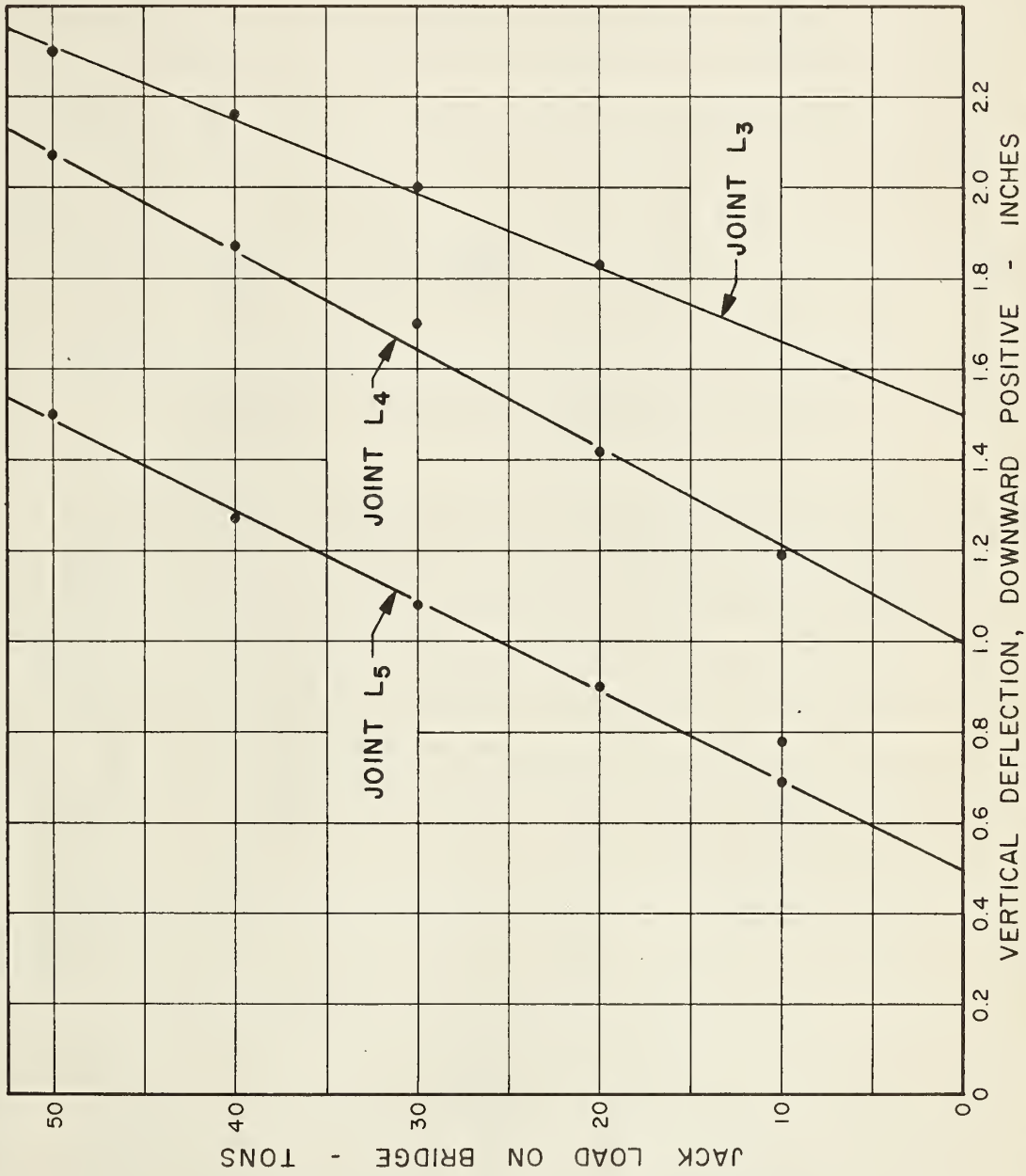
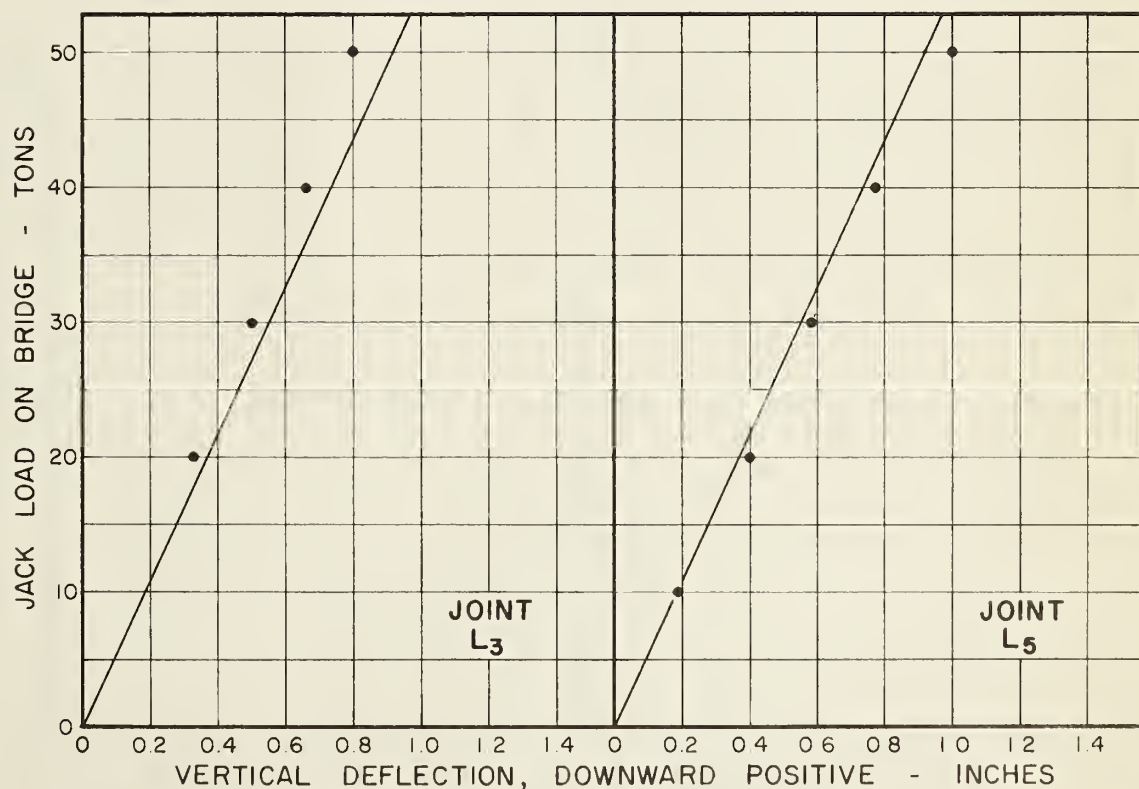
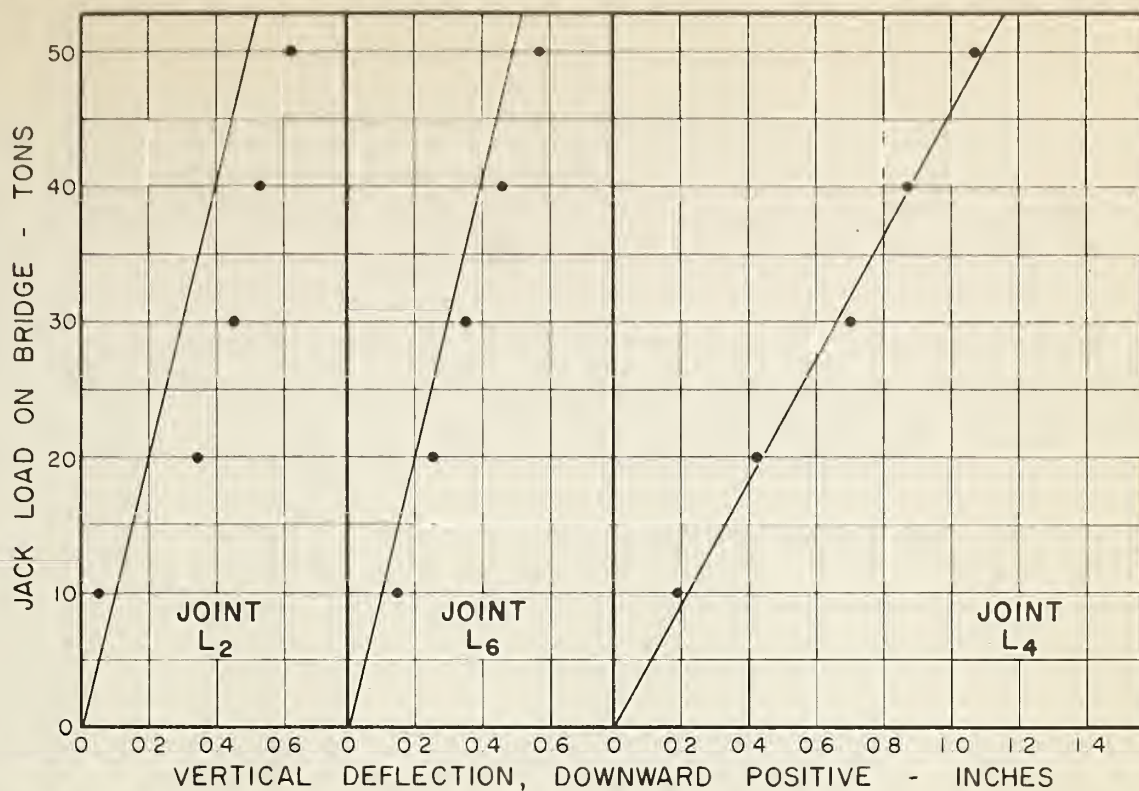
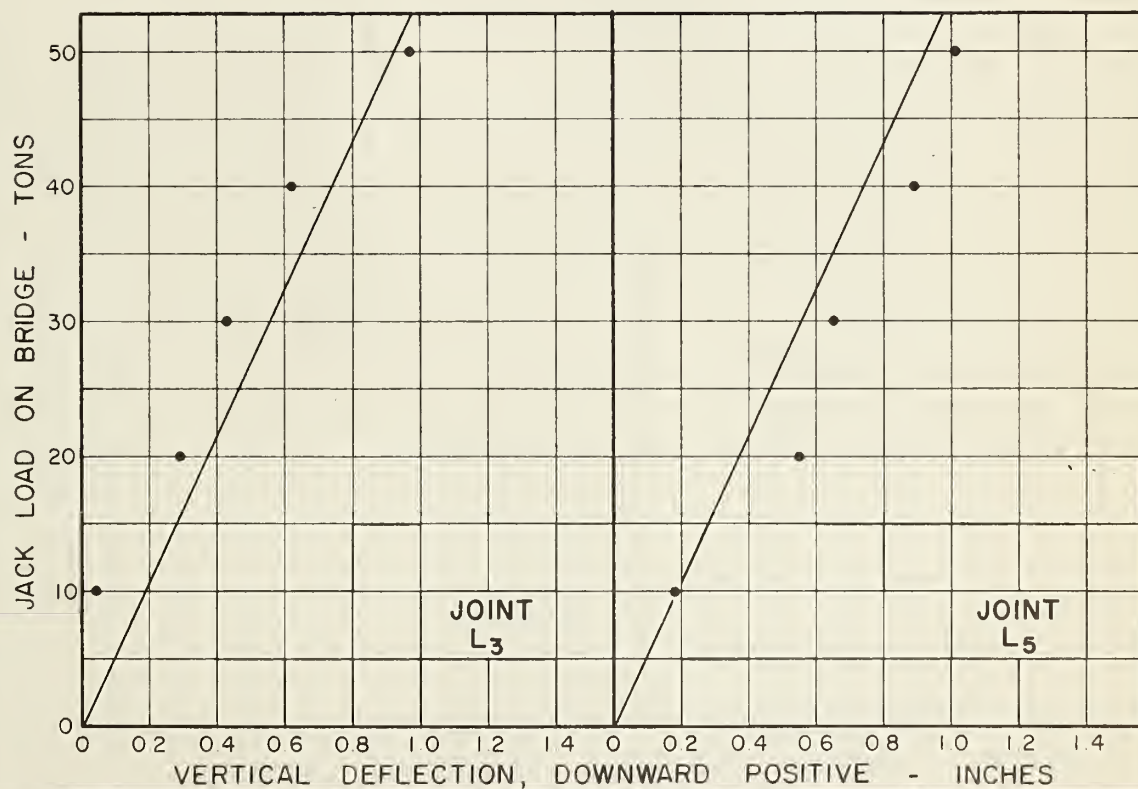
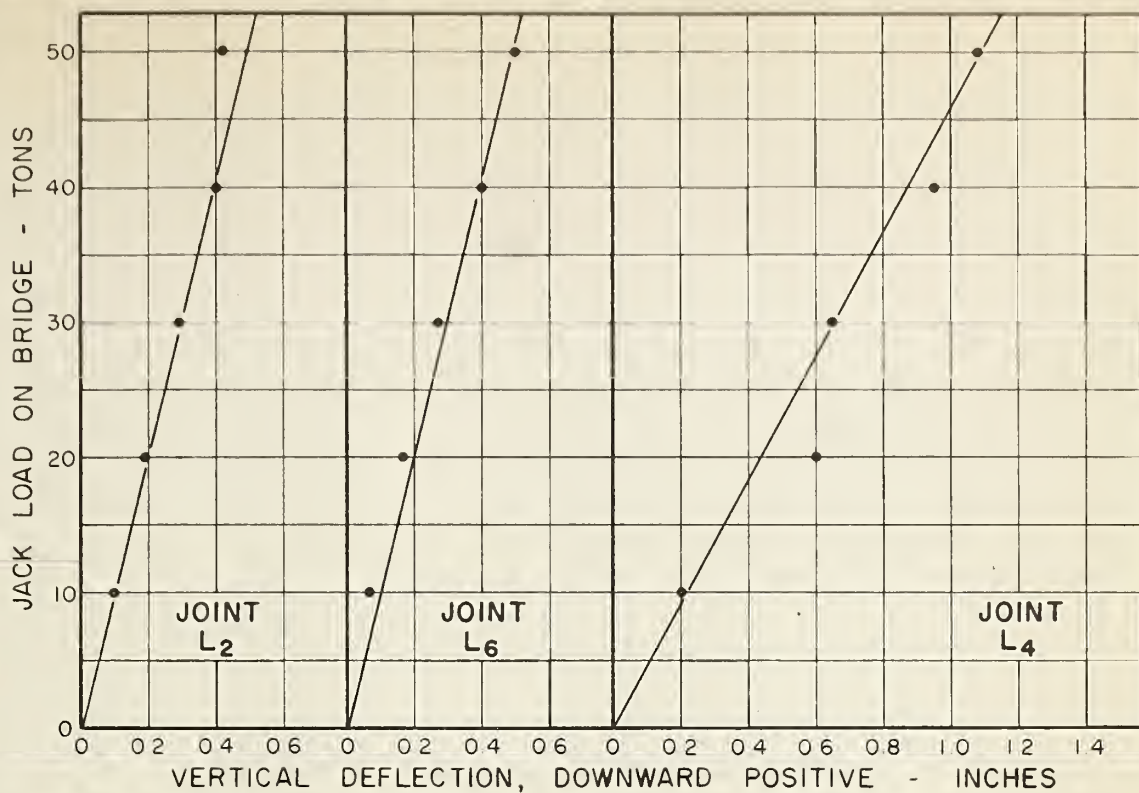


FIGURE 4.3 - CORRECTION DIAGRAM FOR VERTICAL DEFLECTIONS OF NORTH TRUSS
OCTOBER TEST



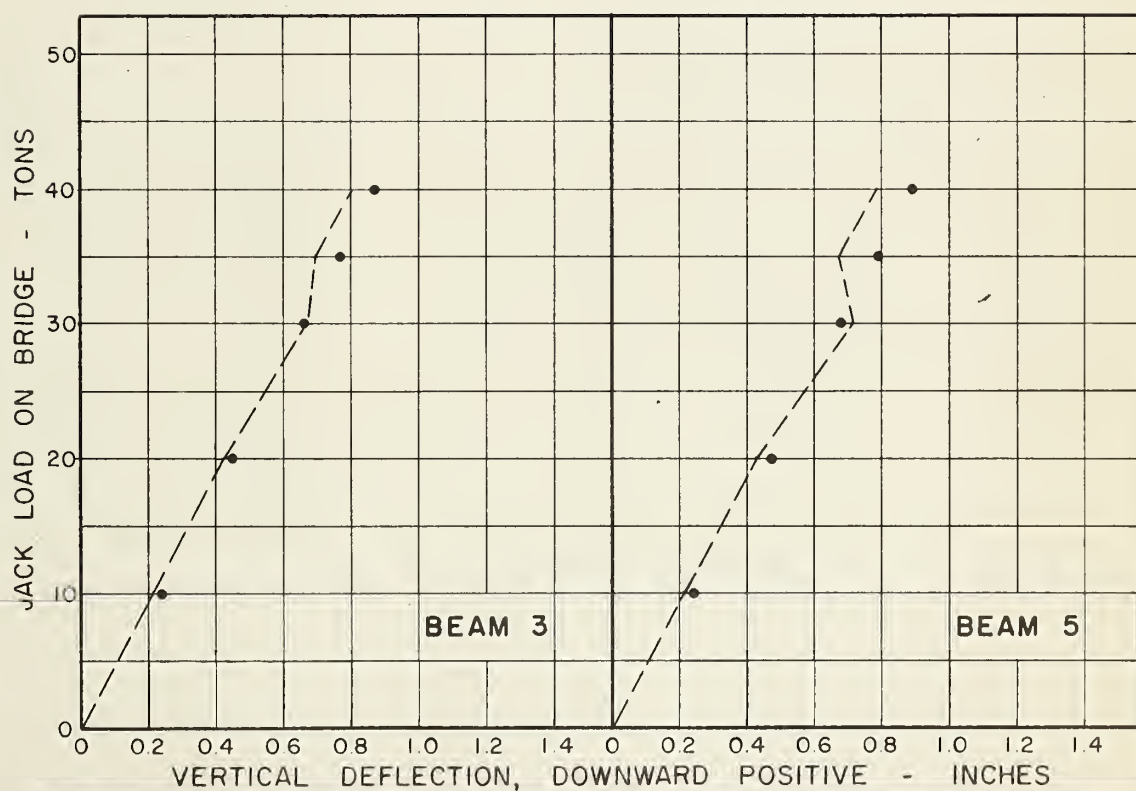
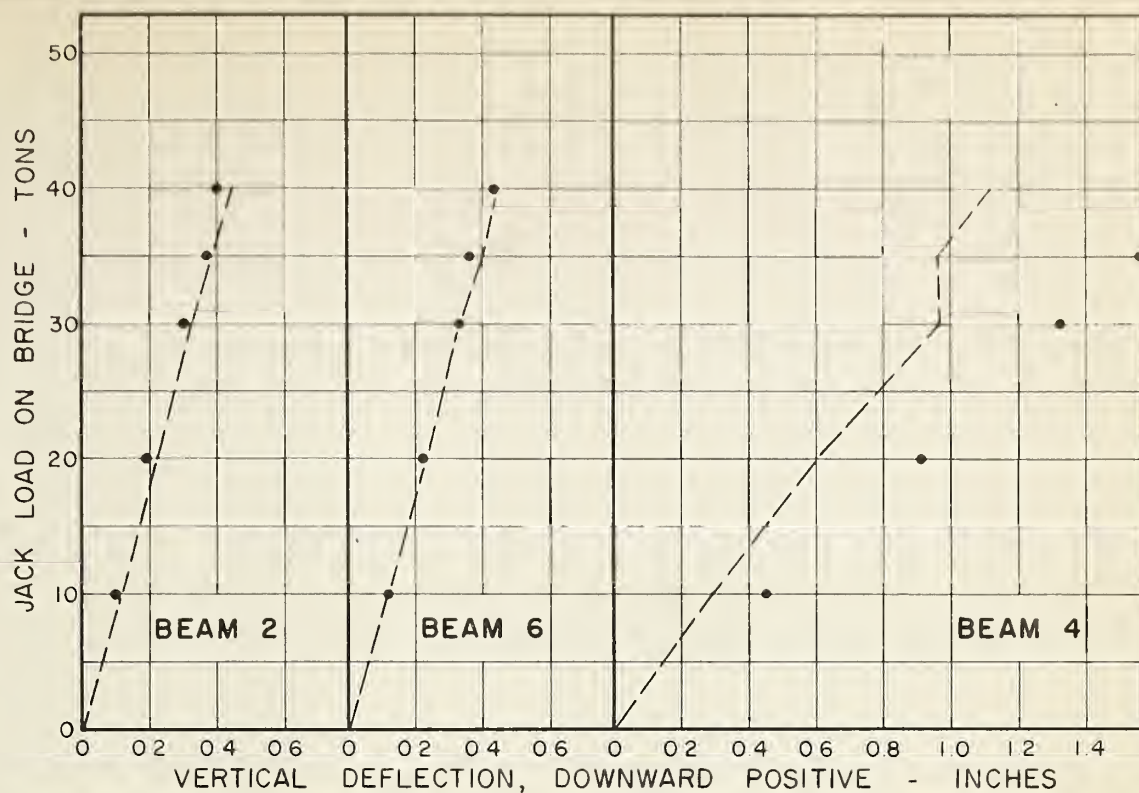
THEORETICAL DEFLECTIONS ILLUSTRATED BY SOLID LINES

FIGURE 4.4 - VERTICAL DEFLECTIONS OF NORTH TRUSS
OCTOBER TEST



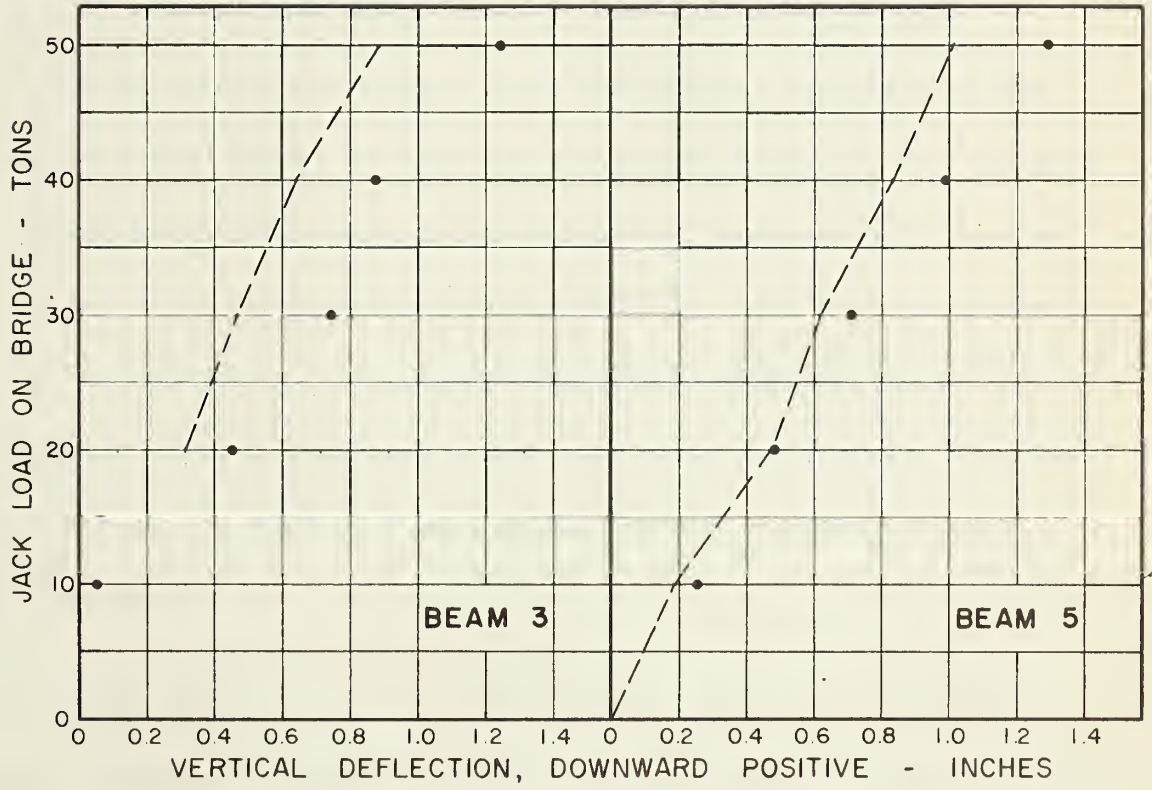
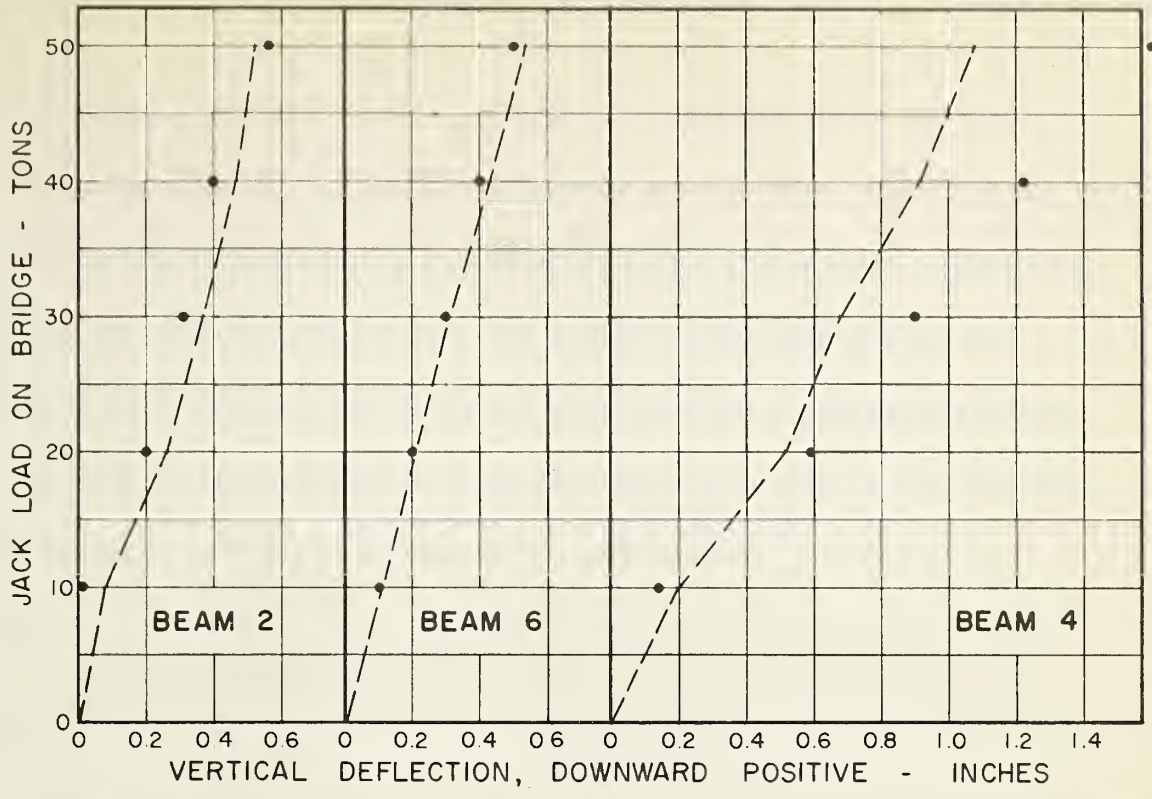
THEORETICAL DEFLECTIONS ILLUSTRATED BY SOLID LINES

FIGURE 4.5 - VERTICAL DEFLECTIONS OF SOUTH TRUSS
OCTOBER TEST



BROKEN LINES ILLUSTRATE AVERAGE DEFLECTIONS OF ENDS OF BEAMS

FIGURE 4.6 - VERTICAL DEFLECTIONS OF FLOOR BEAM CENTERLINES - JULY TEST



BROKEN LINES ILLUSTRATE AVERAGE DEFLECTIONS OF ENDS OF BEAMS

FIGURE 4.7 - VERTICAL DEFLECTIONS OF FLOOR BEAM CENTERLINES - OCTOBER TEST

effects would certainly be detectable elsewhere; for example, the deflection curves for the south truss could be expected to show some corresponding discontinuity, as could the beam center-line deflections. There is no evidence, however, that a shift in the load did occur. Rather it would appear that the steel wire used as a measuring datum was disturbed, an explanation made the more plausible by the fact that the tests were conducted outdoors during the night.

Truss deflections. Figures 4.1, 4.2, 4.4 and 4.5 depict the vertical deflections of the lower chord panel points of the trusses. While the curves are not truly linear, neither do they display evidence of instability in the load range shown. The solid straight lines represent the simple theoretical truss deflections, computed by the method of virtual work. It must be recognized, however, that the true theoretical deflections would differ somewhat, because each truss is warped out of its original plane during loading. This fact notwithstanding, the experimental curves agree reasonably well with the so-called theoretical. This is true even in the case of the north truss, October test (Figure 4.4), further substantiating the remarks concerning the initial readings.

Beam deflections. The deflections of the beam centerlines are shown in Figures 4.6 and 4.7. They have been measured relative to a fixed datum, and consequently include the effects of the truss deflections.

The curves for the July test display marked linearity, while those for the October test show the same erratic pattern which characterizes the corresponding truss deflections. The broken lines represent the average of the north and south truss deflections at the same panel point, and hence the deflection of the beam centerline if no moment were applied to the beam. Thus the departure, if any, of the actual deflections from these lines is a rough measure of the bending moment induced in the beam. On this basis, it would appear that beams two, three, five and six in the July test, and beams two and six in the October test, carried virtually no load. It should be appreciated that the severe discontinuities appearing in the broken lines of Figure 4.6 simply manifest the effect of the recorded July deflections of the north truss. An explanation of the latter has already been advanced; the linearity of the beam deflections seems to corroborate this explanation.

II. WEB MEMBERS

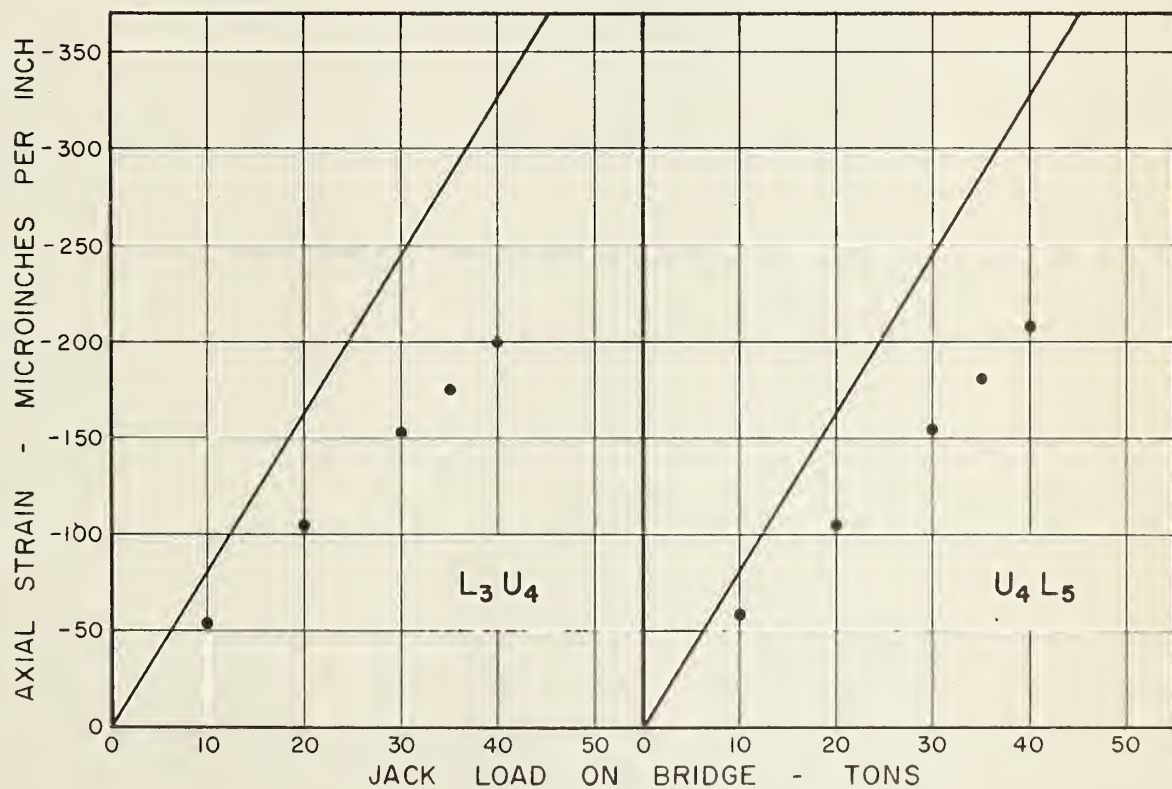
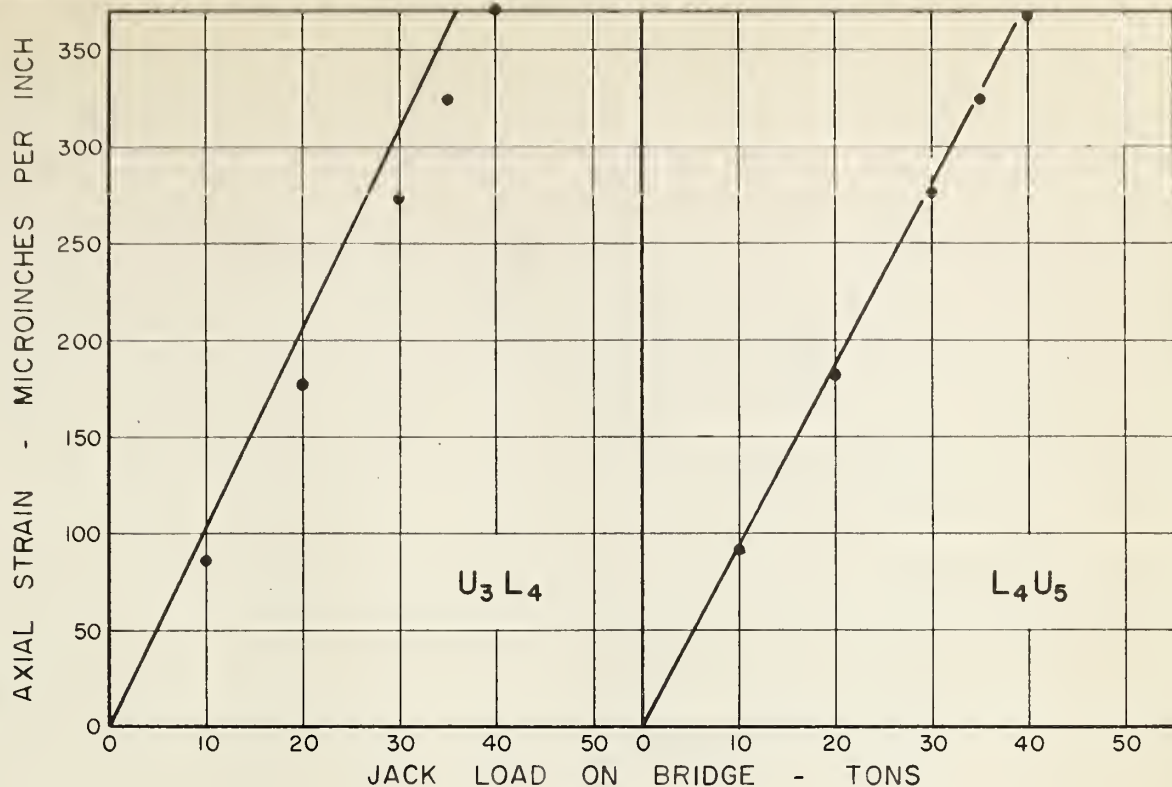
The classification "web members" includes here the diagonal and vertical members of the truss, including the end diagonals. These two types of member will be discussed independently, since the action of the vertical members of a pony-truss is quite distinct from that of the diagonals.

Diagonal Members. The end and first interior diagonals of the truss consisted of double angle members, and the remaining diagonals

of single angles.

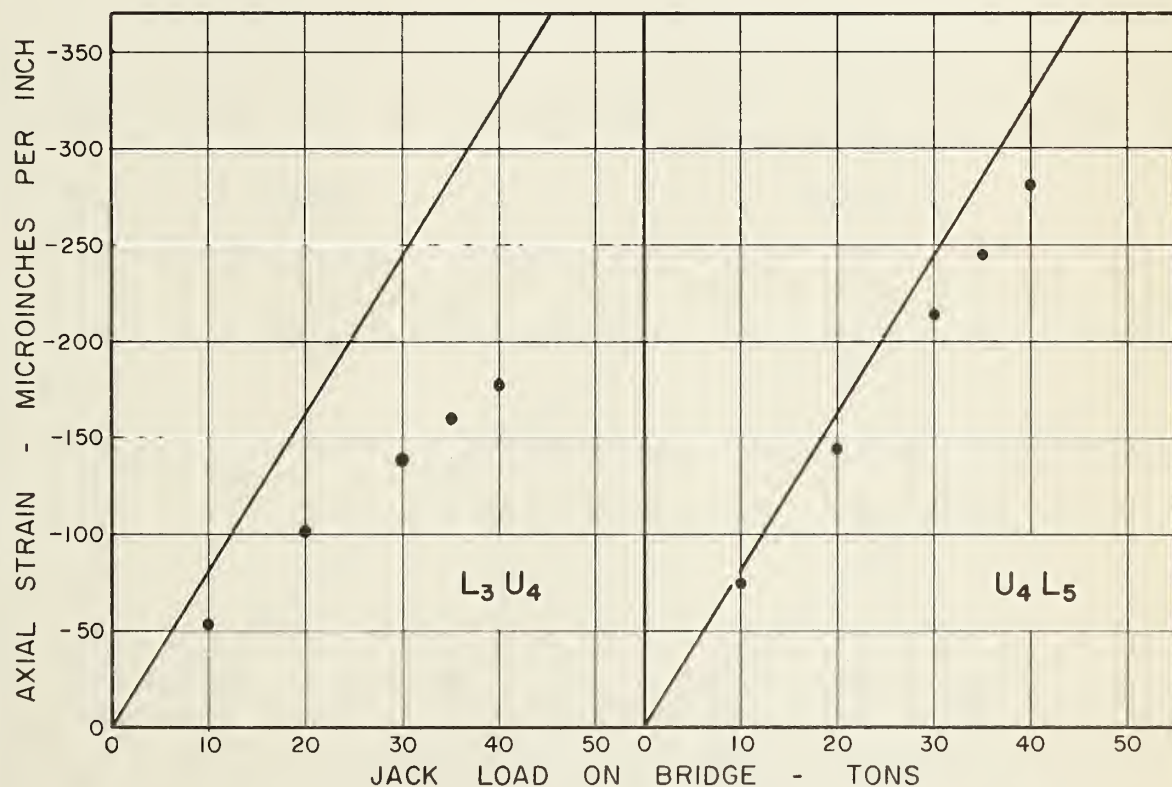
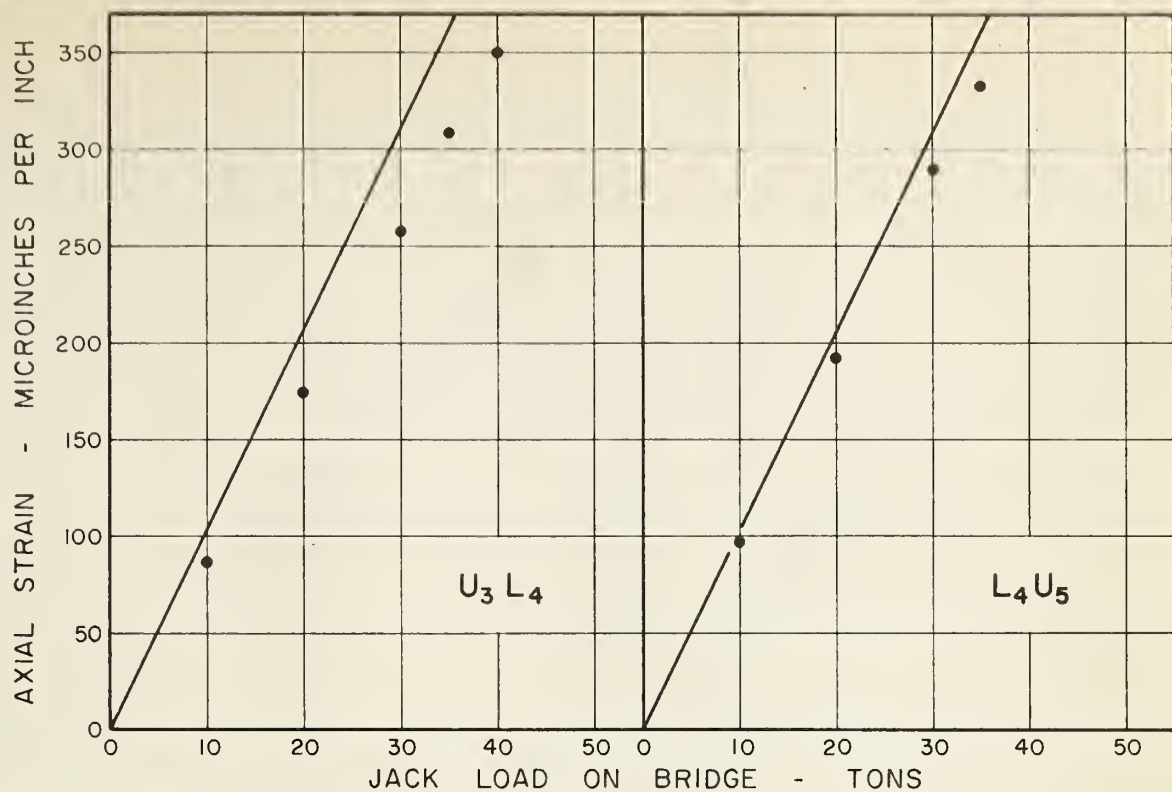
Of the experimental data accumulated during the tests, that relating to the double angle diagonal was least satisfactory. Lukawitski's results⁽⁸⁾ indicated that the axial strain in such a member could be found by merely averaging the strain measurements taken as outlined in Chapter III. Because of this, no change was made in the strain gauge positions for the present tests. Subsequent analysis of the results of these tests, however, suggested that the simple average of the strain readings does not represent the axial strain. The total load applied to the truss, as determined from the so-called axial strains in the diagonals, was found to be in disagreement with the actual applied load. Consequently, a double angle member was tested in tension at the University of Alberta, the results confirming that it is impossible to obtain the axial strain in such a member using strain gauges in the manner in which they were used on the bridge. Since this part of the test data has thereby been rendered useless, it has not been presented herein.

Lukawitski's treatment of the single angle, however, is eminently successful, at least for tension members. His method of measuring the axial strain was verified by the author in a laboratory tension test. Accordingly, the axial strains in the single angle diagonal have been presented in Figures 4.8 through 4.11. In each diagram, the solid straight line indicates the theoretical load-strain relationship.



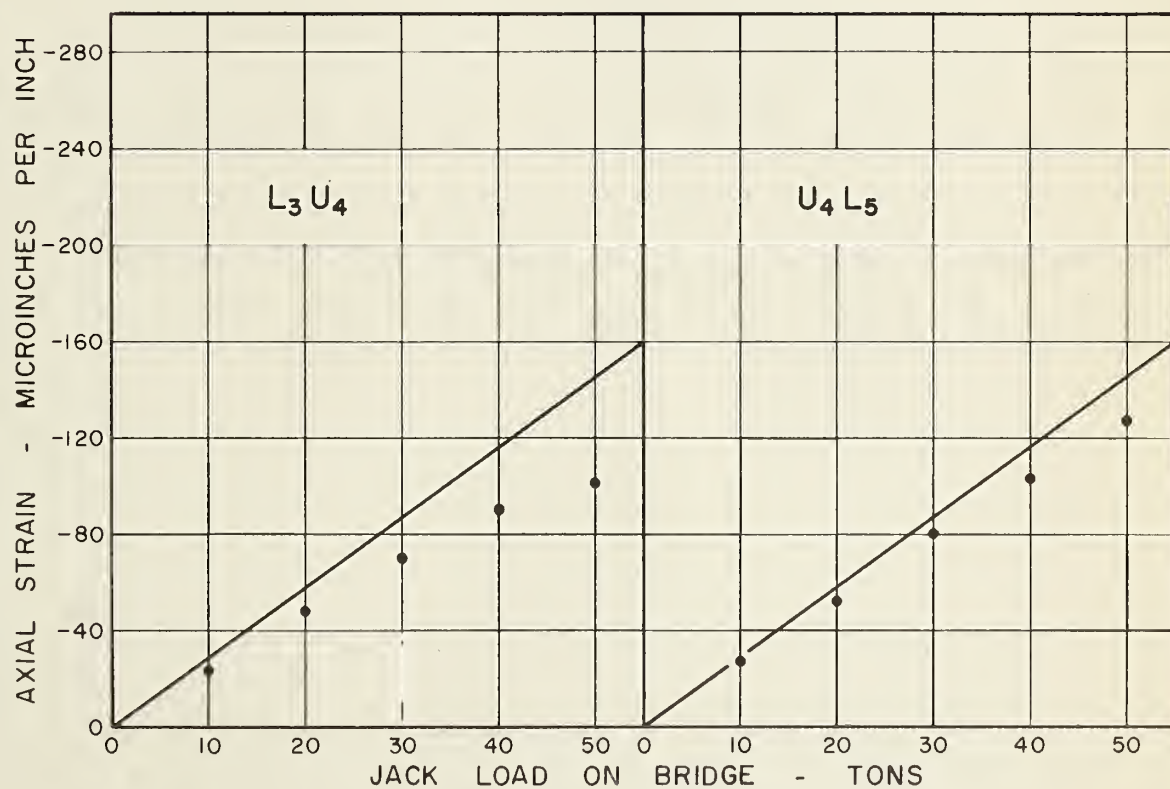
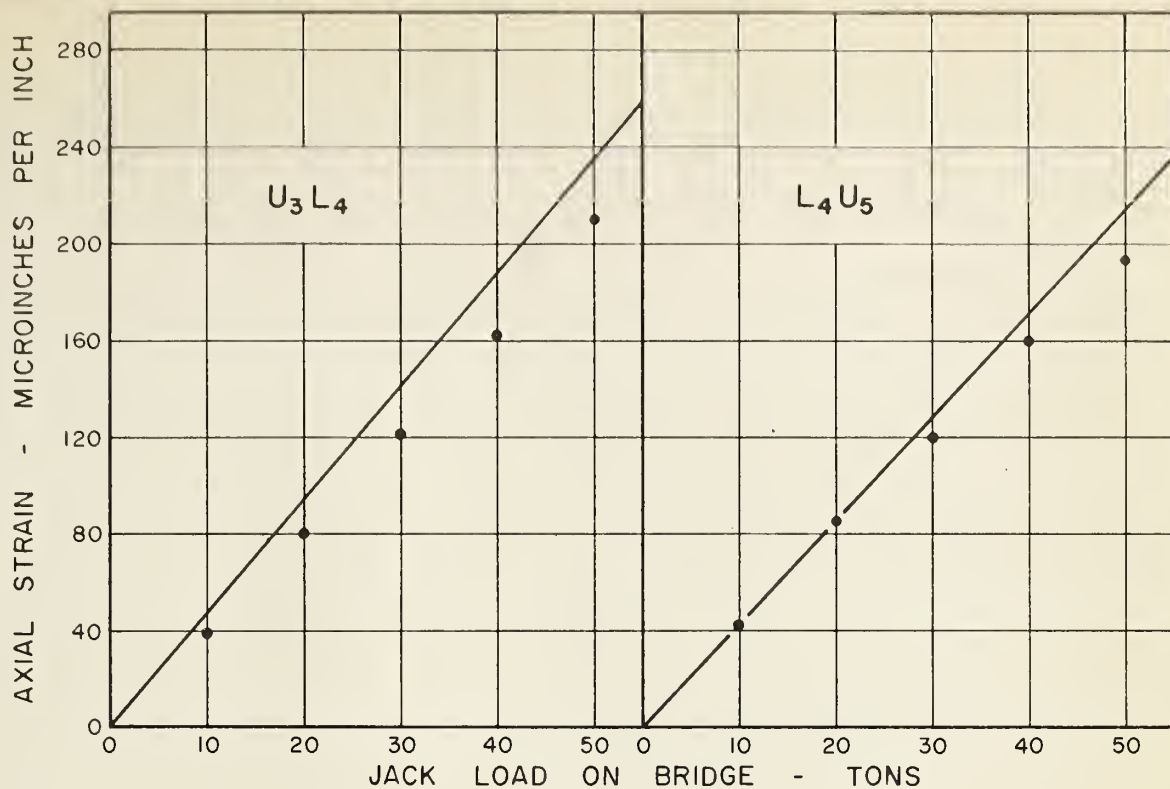
THEORETICAL STRAINS ILLUSTRATED BY SOLID LINES

FIGURE 4.8 - AXIAL STRAINS IN SINGLE ANGLE MEMBERS
NORTH TRUSS - JULY TEST



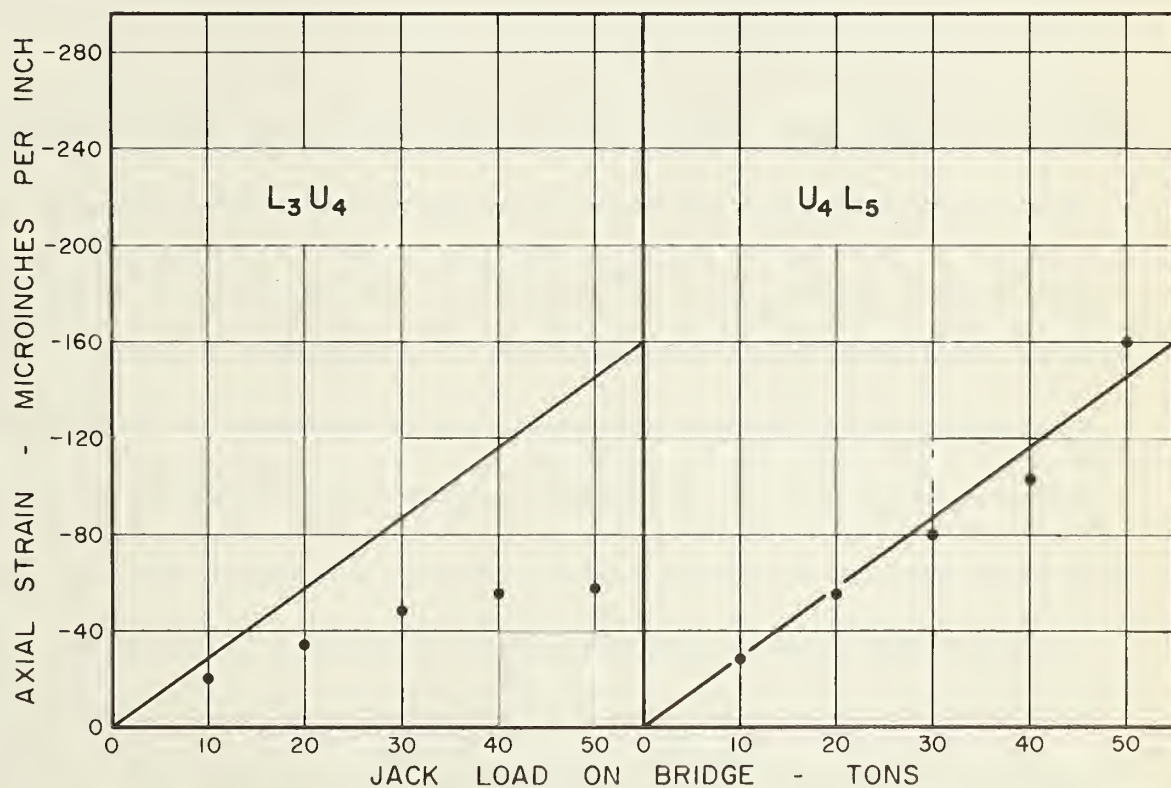
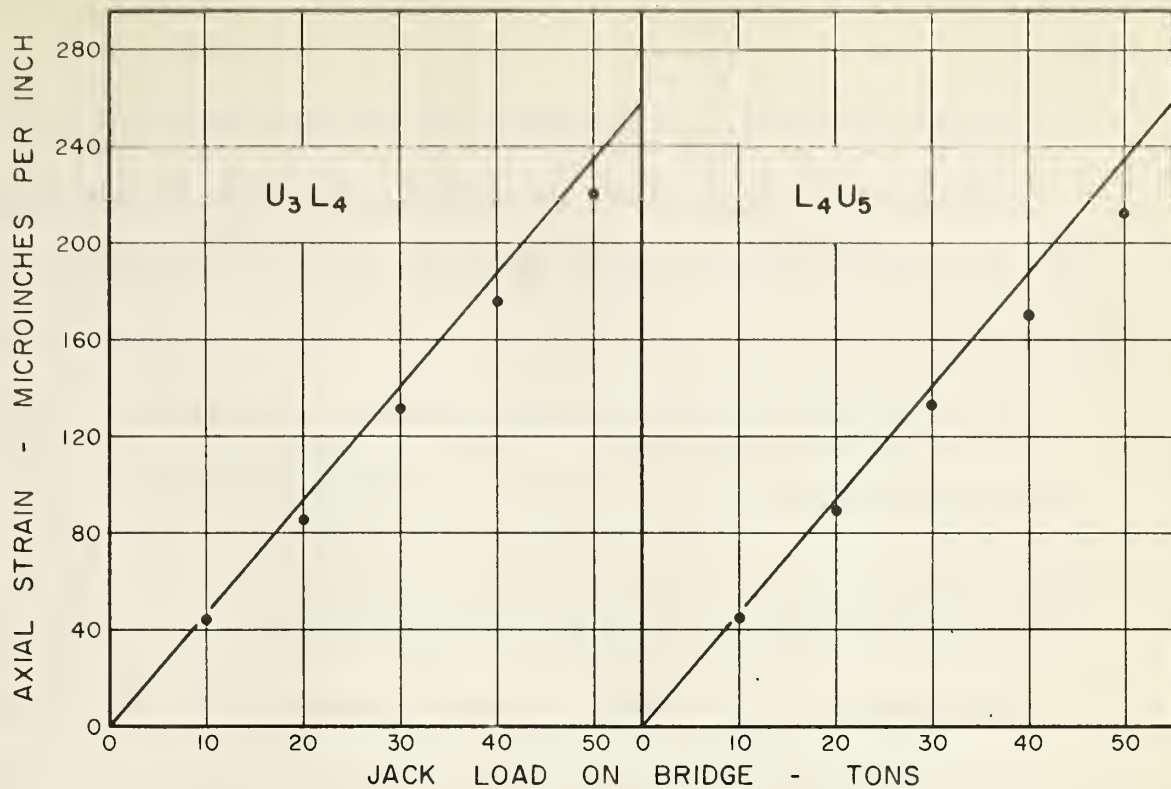
THEORETICAL STRAINS ILLUSTRATED BY SOLID LINES

FIGURE 4.9 - AXIAL STRAINS IN SINGLE ANGLE MEMBERS
SOUTH TRUSS - JULY TEST



THEORETICAL STRAINS ILLUSTRATED BY SOLID LINES

FIGURE 4.10 - AXIAL STRAINS IN SINGLE ANGLE MEMBERS
NORTH TRUSS - OCTOBER TEST



THEORETICAL STRAINS ILLUSTRATED BY SOLID LINES

FIGURE 4.11 - AXIAL STRAINS IN SINGLE ANGLE MEMBERS
SOUTH TRUSS - OCTOBER TEST

It will be noted that the measured strains almost without exception fall short of the theoretical, although in many cases the departure is slight. In general, the tension member strains conform more closely to the theoretical. Whether this indicates an actual dropping off of the load in the compression members, or whether the method of determining the axial strain is not valid for compression members, cannot be rigorously demonstrated. It is not unreasonable to suspect that the rate of axial strain in the compression diagonals did decrease as they became bowed. Under such circumstances, however, the rate of axial strain in the tension diagonals could be expected to increase. Such increase is not apparent.

Attention is now directed to the fact that the summation of the vertical components of the forces in the diagonal web members adjacent to any panel point of the bridge must equal the external vertical load applied to that panel point. Applying this law of statics, it is obvious from Figures 4.8 through 4.11 that the center panel point load, as thus determined, is exceeded by the actual load applied to the bridge deck at the center panel point. Hence, if the axial strains shown in these diagrams are accepted as correct, then a portion of the load must have been transferred, presumably by the decking, to other panel points. On the other hand, it is difficult to conceive that, in the event of load transfer, the tension members would not show a decrease in stress proportional to that shown by the compression members.

This conflict might be resolved if the other panel points of the bridge were similarly analyzed. All other panel points involved double angle members, however, and it has been noted previously that reliable information concerning such members is not available.

Vertical members. Whereas the diagonal member is of interest only in terms of its axial stress, the vertical member fulfills an important function in its role of providing lateral support to the top chord.

As the top chord deflects laterally, bending stresses are induced in the vertical. Furthermore, since the vertical acts integrally with the floor beam, bending of the latter also produces bending stresses in the vertical, if it is restrained at the top. Lukawitski has pointed out⁽⁸⁾ that, in the absence of external loads applied to the floor beam, a linear relation should prevail between the lateral deflection of the top of the vertical, and the bending moment at any given point on the vertical. It follows, then, that the moment-deflection diagram for a given vertical should indicate, by reason of its linearity or non-linearity, the absence or presence, respectively, of external load on the attached floor beam. Moreover, there should exist a so-called spring constant for each cross-frame where the floor beam is not loaded. Lukawitski has considered one-half of the cross-frame as a cantilever, assuming the floor beam to be fixed in bending at its point of zero rotation, that is, at the centerline of the cross-

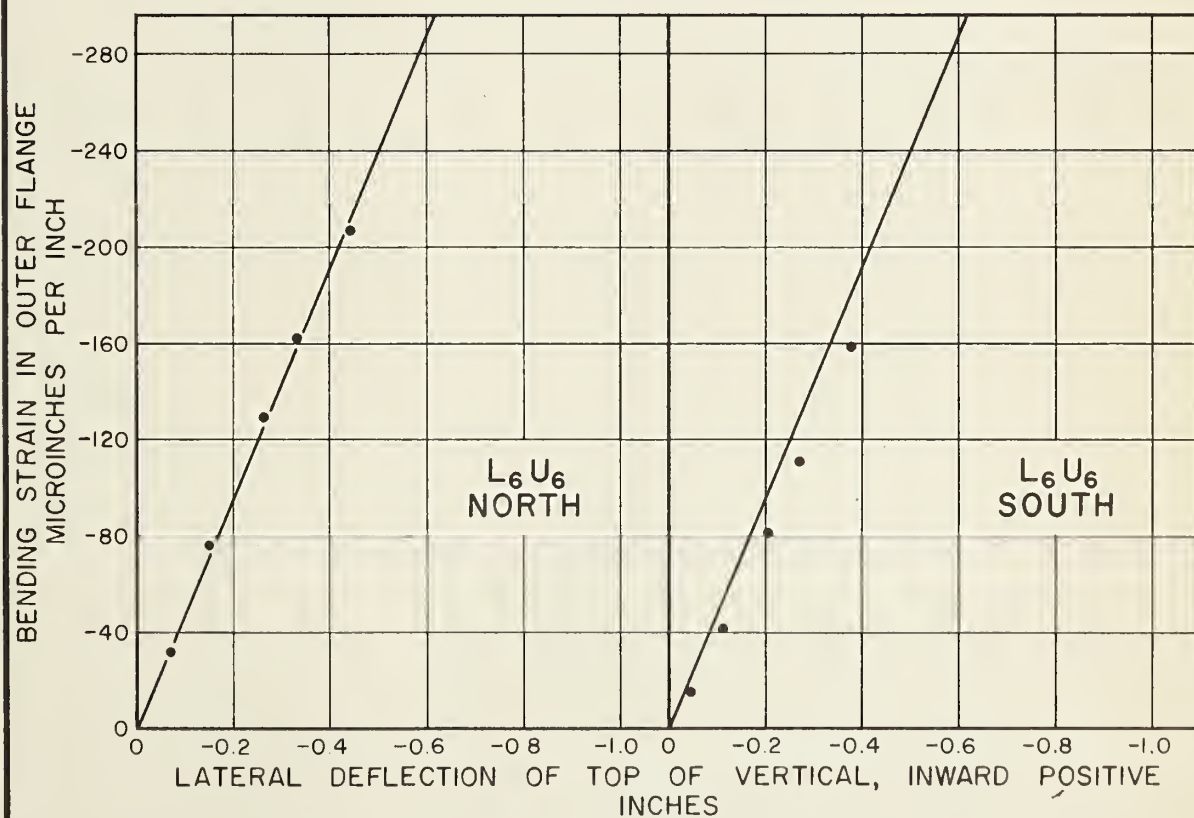
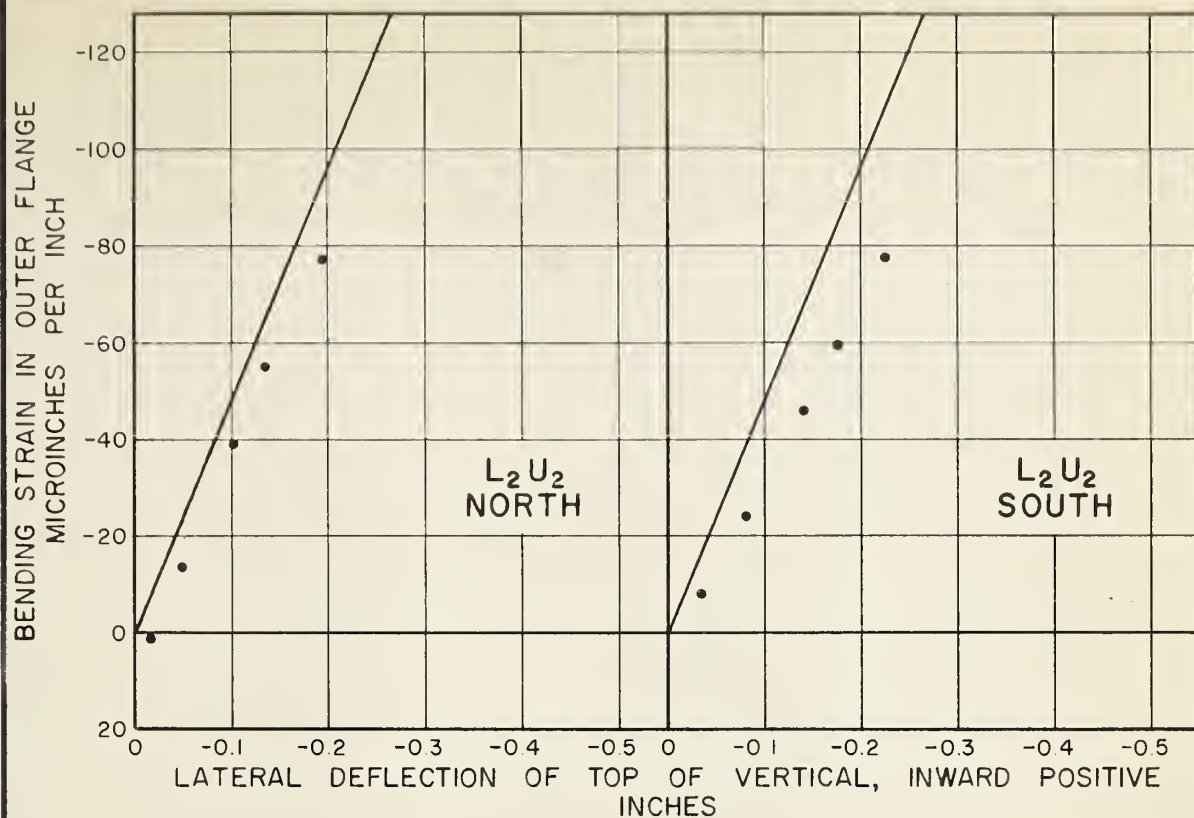
frame. The spring constant, defined as the load required to deflect the top of the vertical through a distance of one inch in the plane of the cross-frame, thus determined has a value of 2.16 kips per inch.

Figures 4.12 through 4.17 show the bending-lateral deflection relationships for the vertical members. The solid straight lines represent Lukawitski's theoretical spring constant. Figures 4.12, 4.13, and 4.15 pertain to those verticals whose connecting floor beams were theoretically not loaded during the tests. Marked linearity may be generally observed among these diagrams, as well as fairly good agreement with the theoretical lines. This becomes particularly apparent when Figures 4.14, 4.16, and 4.17 are examined. These curves display considerable departure both from linearity, and from the theoretical.

Although it is recognized that the above-mentioned diagrams are but a qualitative indication of the presence or absence of external load on the floor beams, it is felt that there is strong evidence of the validity of Lukawitski's argument.

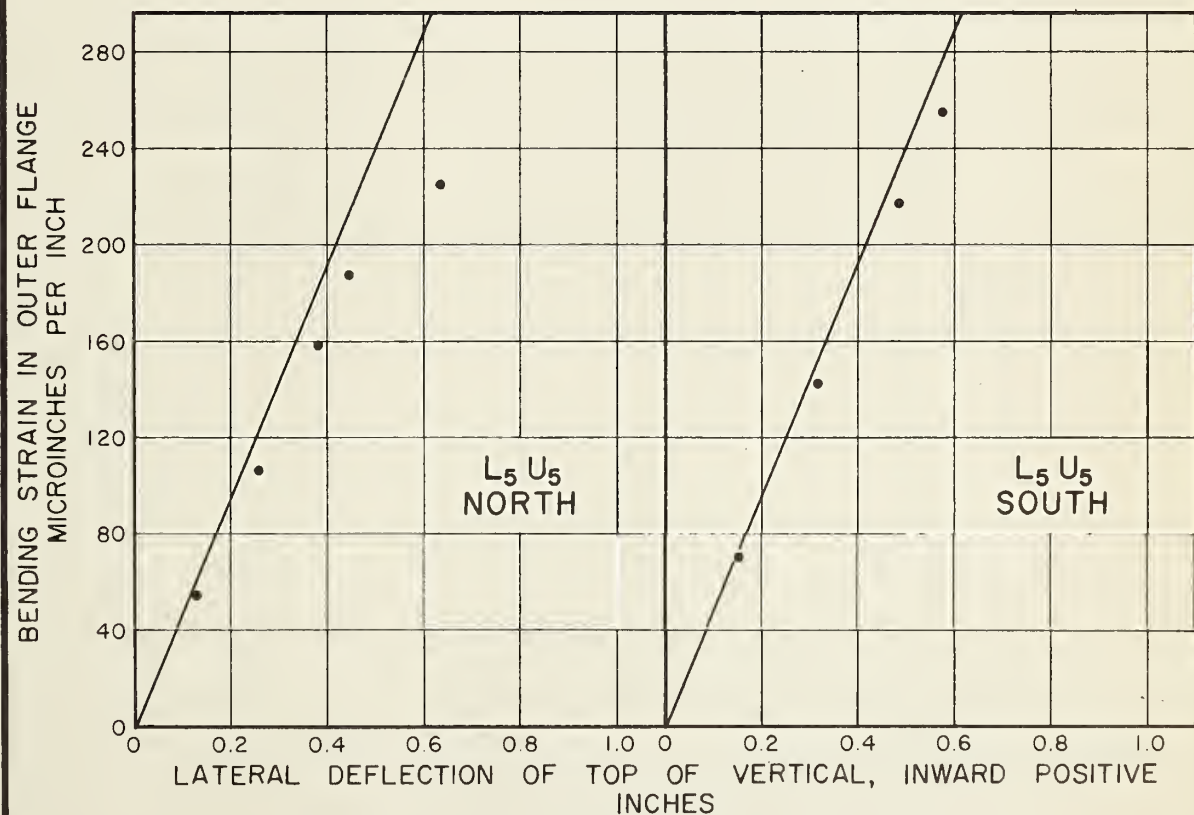
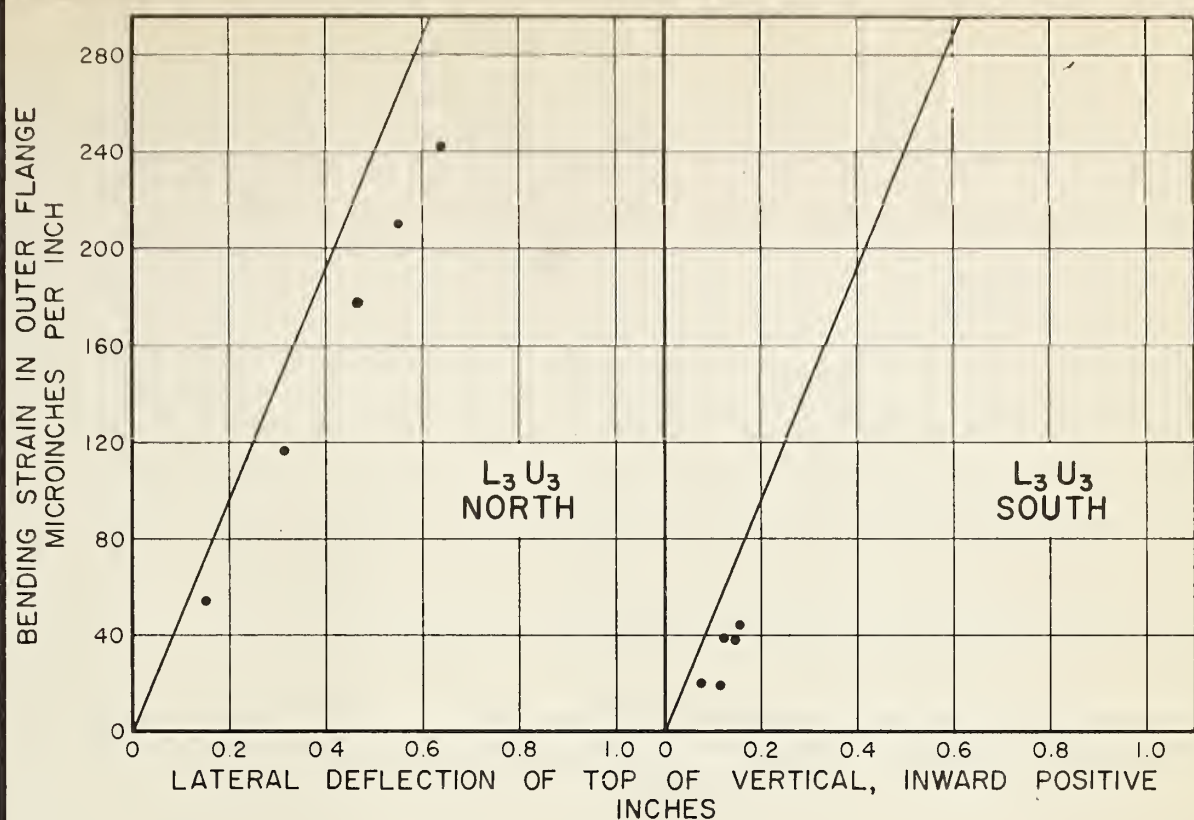
III. FLOOR BEAMS

With the exception of the center floor beam during the July test, strain readings were taken at five locations along the length of each beam. In the excepted case, the station at the center-line of the beam was inadvertently omitted.



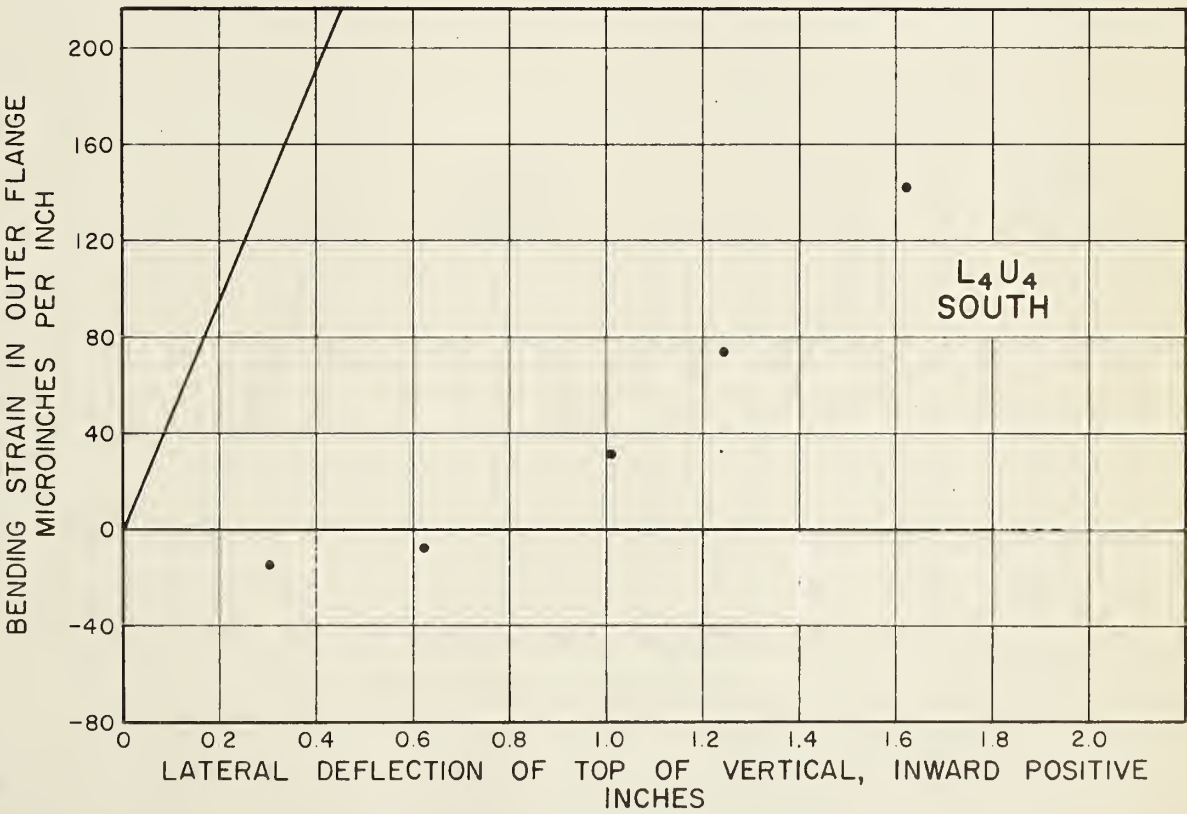
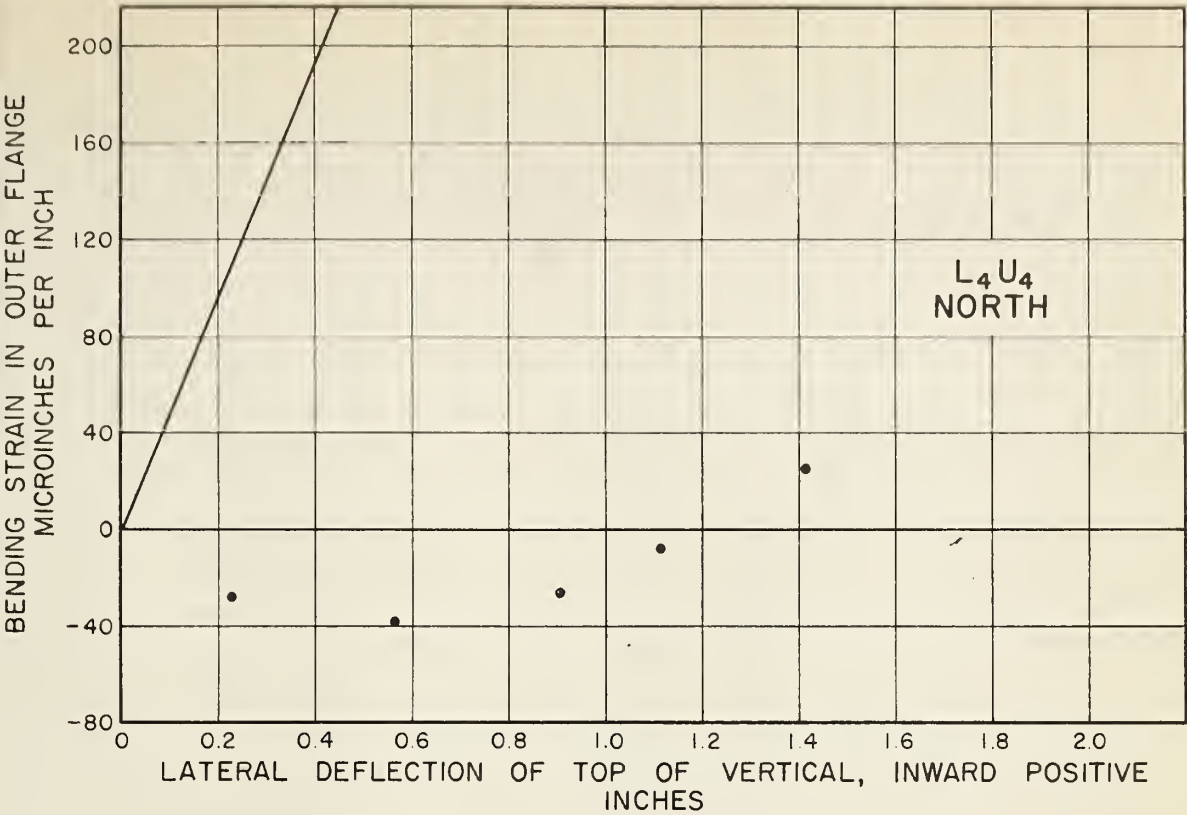
SOLID LINES ILLUSTRATE THEORETICAL: SEE TEXT

FIGURE 4.12 - VERTICAL MEMBERS: BENDING STRAINS
VERSUS LATERAL DEFLECTIONS - JULY TEST



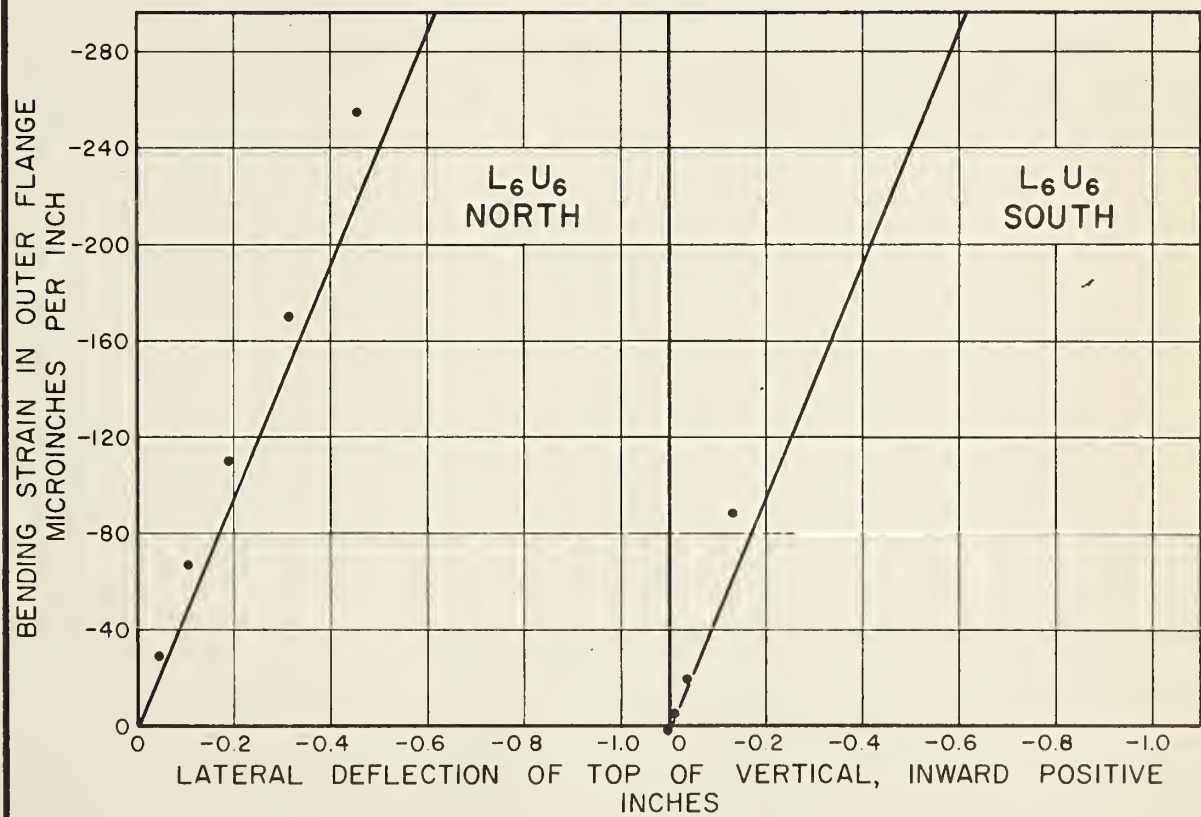
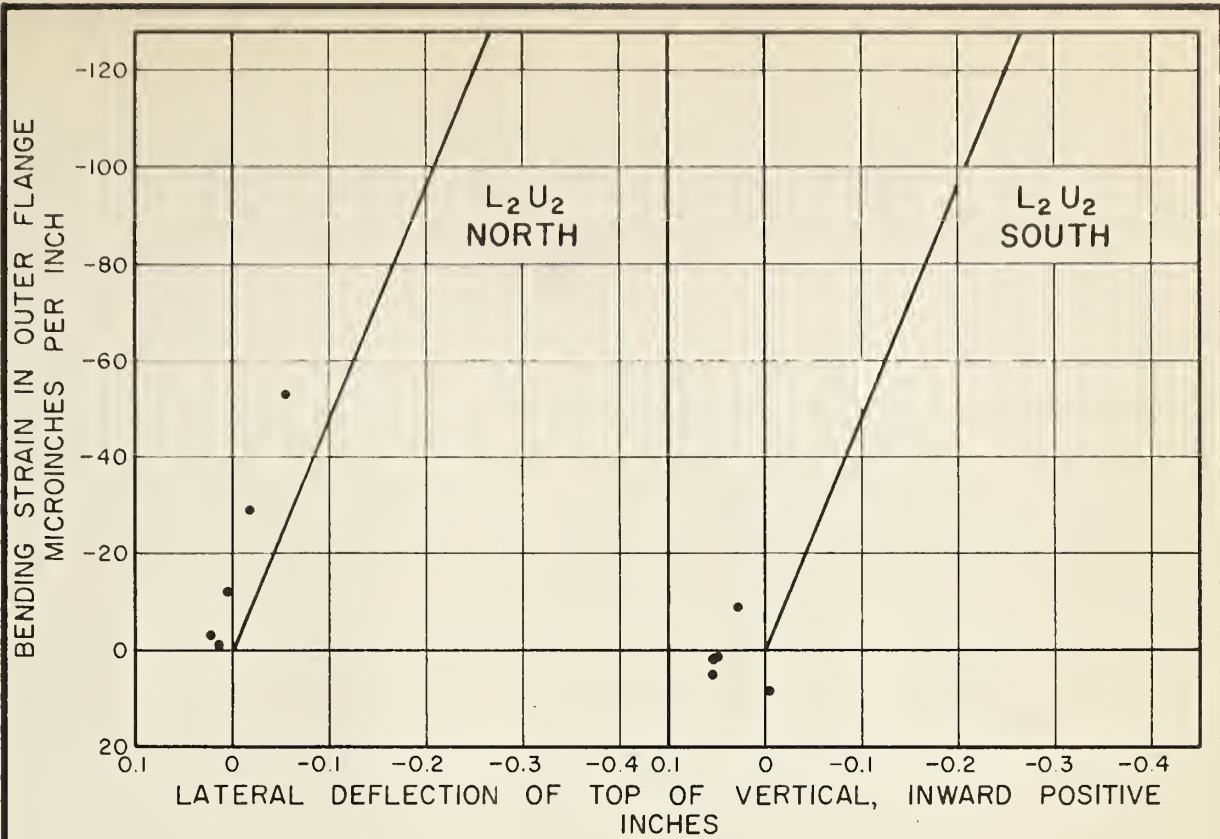
SOLID LINES ILLUSTRATE THEORETICAL: SEE TEXT

FIGURE 4.13 - VERTICAL MEMBERS: BENDING STRAINS VERSUS LATERAL DEFLECTIONS - JULY TEST



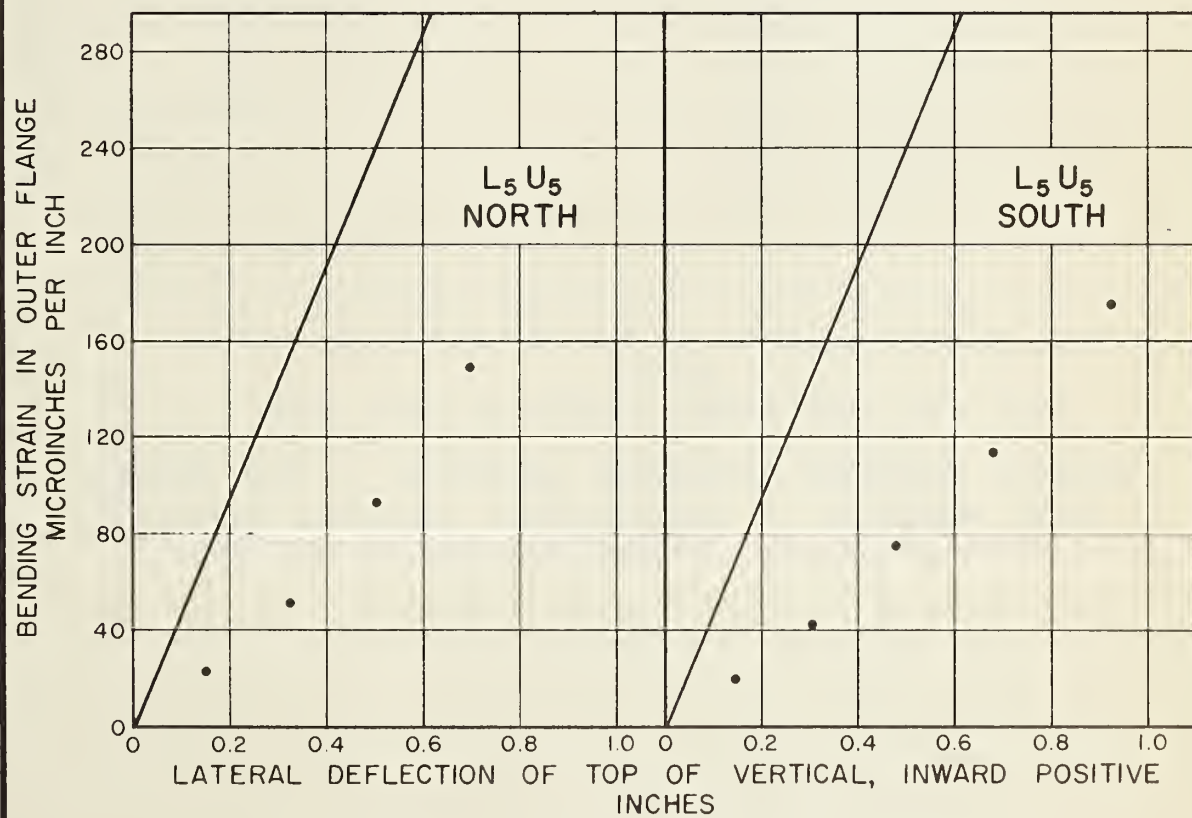
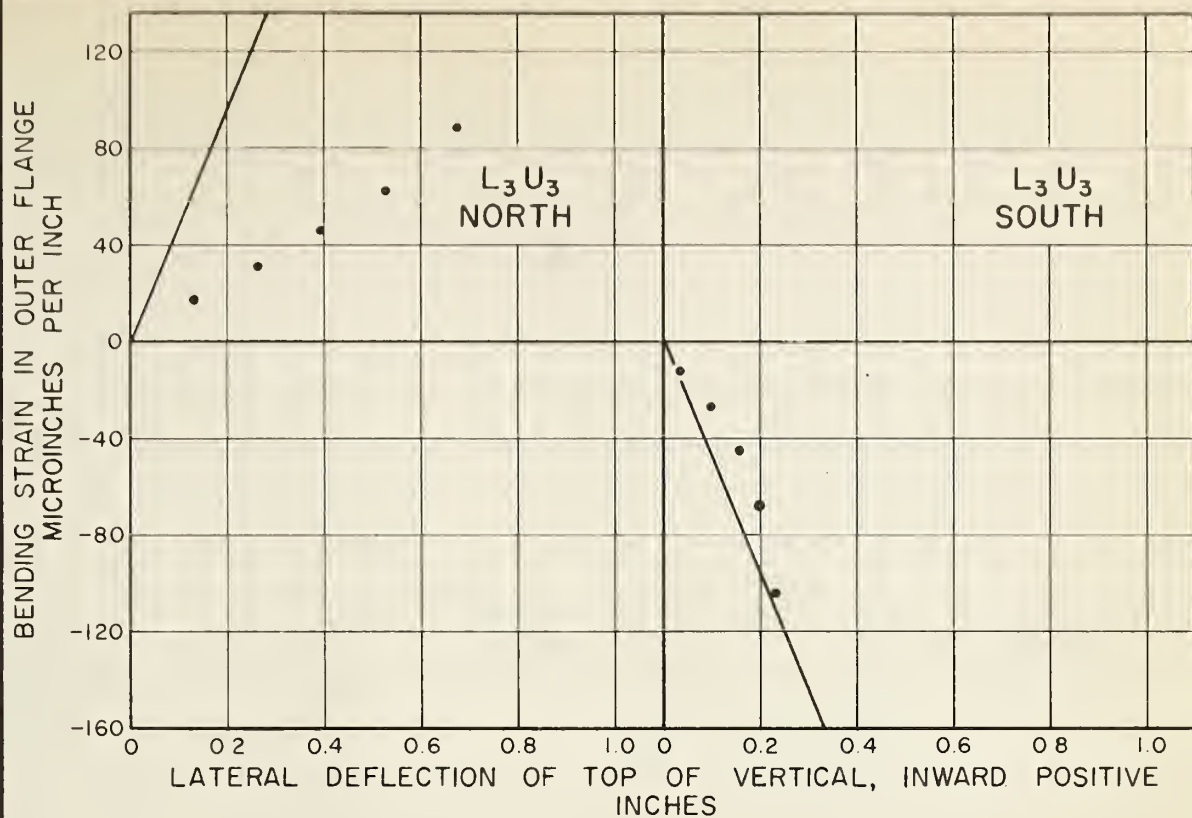
SOLID LINES ILLUSTRATE THEORETICAL: SEE TEXT

FIGURE 4.14 - VERTICAL MEMBERS: BENDING STRAINS
VERSUS LATERAL DEFLECTIONS - JULY TEST



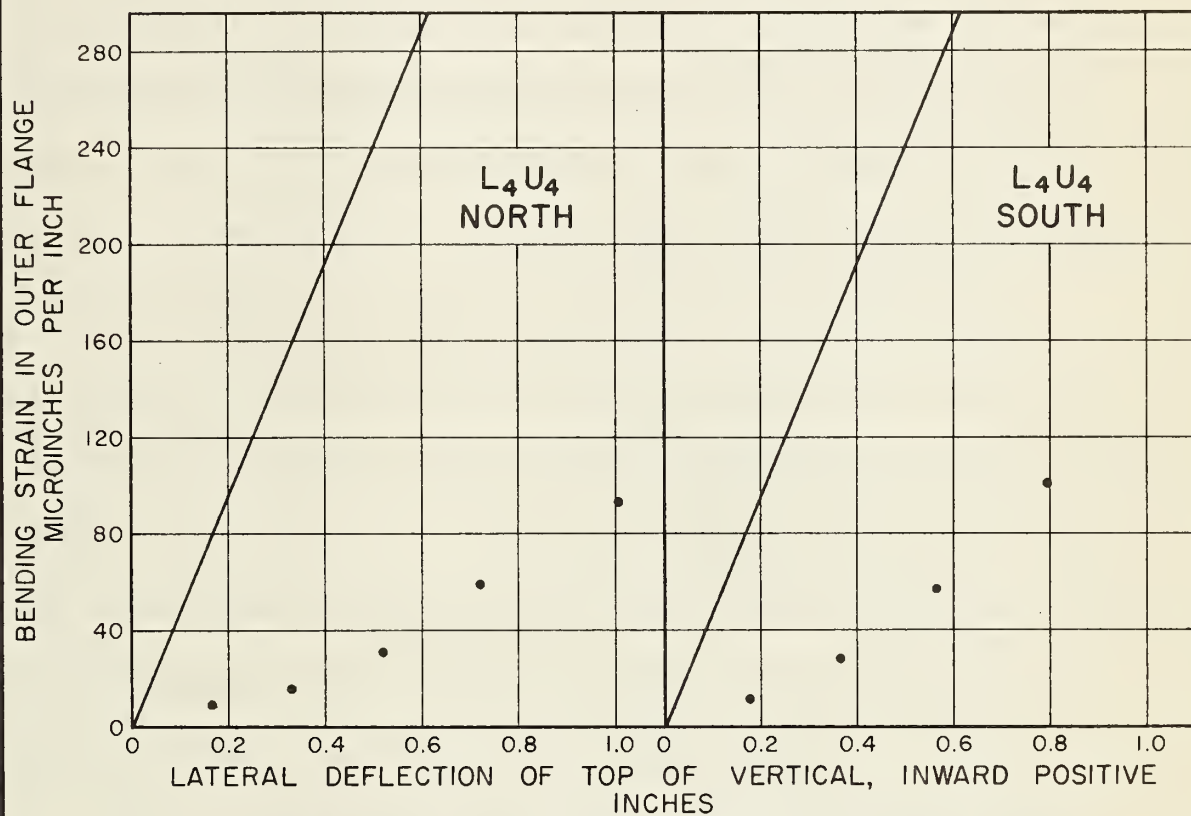
SOLID LINES ILLUSTRATE THEORETICAL: SEE TEXT

FIGURE 4.15 - VERTICAL MEMBERS: BENDING STRAINS
VERSUS LATERAL DEFLECTIONS - OCTOBER TEST



SOLID LINES ILLUSTRATE THEORETICAL: SEE TEXT

FIGURE 4.16 - VERTICAL MEMBERS: BENDING STRAINS
VERSUS LATERAL DEFLECTIONS - OCTOBER TEST



SOLID LINES ILLUSTRATE THEORETICAL: SEE TEXT

FIGURE 4.17 - VERTICAL MEMBERS: BENDING STRAINS
VERSUS LATERAL DEFLECTIONS - OCTOBER TEST

The results of these strain readings appear to be quite reliable. Such deficiencies as exist are the result not of inaccuracy, but of an insufficient number of strain gauge stations on each beam.

Distribution of load on an individual beam. In the early stages of the analysis of the experimental data, it was hoped that information concerning the distribution of load on a floor beam could be derived from the slope of the measured bending moment diagram, through successive differentiations. Lacking strain measurements at the ends of the floor beams, the bending moments in the verticals were extrapolated, assuming the latter members to be simple cantilevers, to provide end moments. This procedure yielded seven points on each bending moment curve. By means of an electronic computer, a sixth order polynomial was fitted to these points, and the resulting equations were differentiated twice, with the machine printing coordinates for both the shearing force and the loading diagrams.

The shearing force at the end of a beam, and hence the total load on the beam, is extremely sensitive to the end slope of the bending moment diagram. This slope proved, in the test results, to be extremely sensitive, in turn, to the extrapolated end moment in the beam. Examination of the experimental bending moment diagrams, typical of which are those shown in Figures 4.18 and 4.19, showed that even a small error in the extrapolated end moment could produce a serious error in the end slope of the moment diagram. Furthermore,

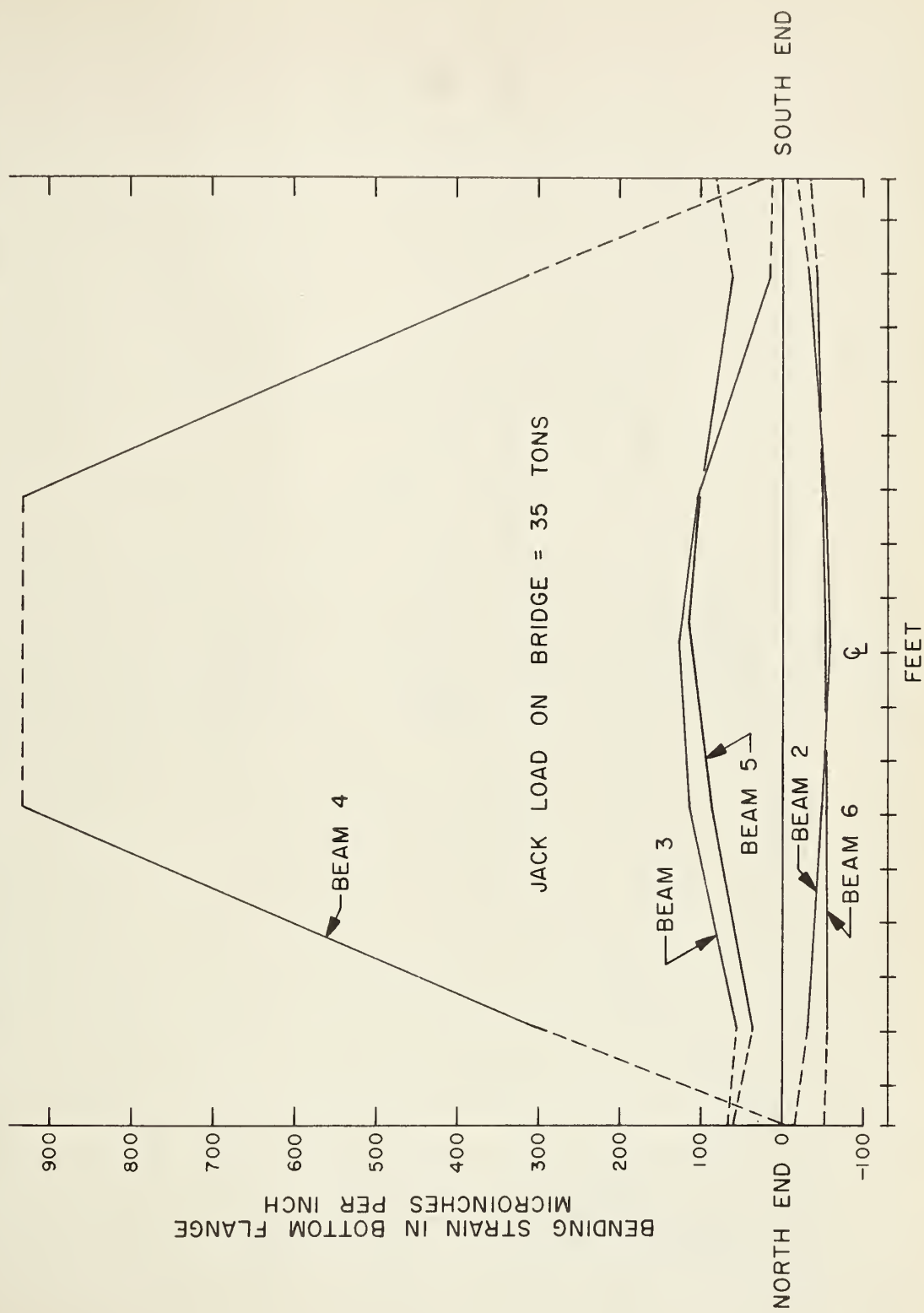


FIGURE 4.18 - TYPICAL FLOOR BEAM BENDING STRAIN DIAGRAMS - JULY TEST

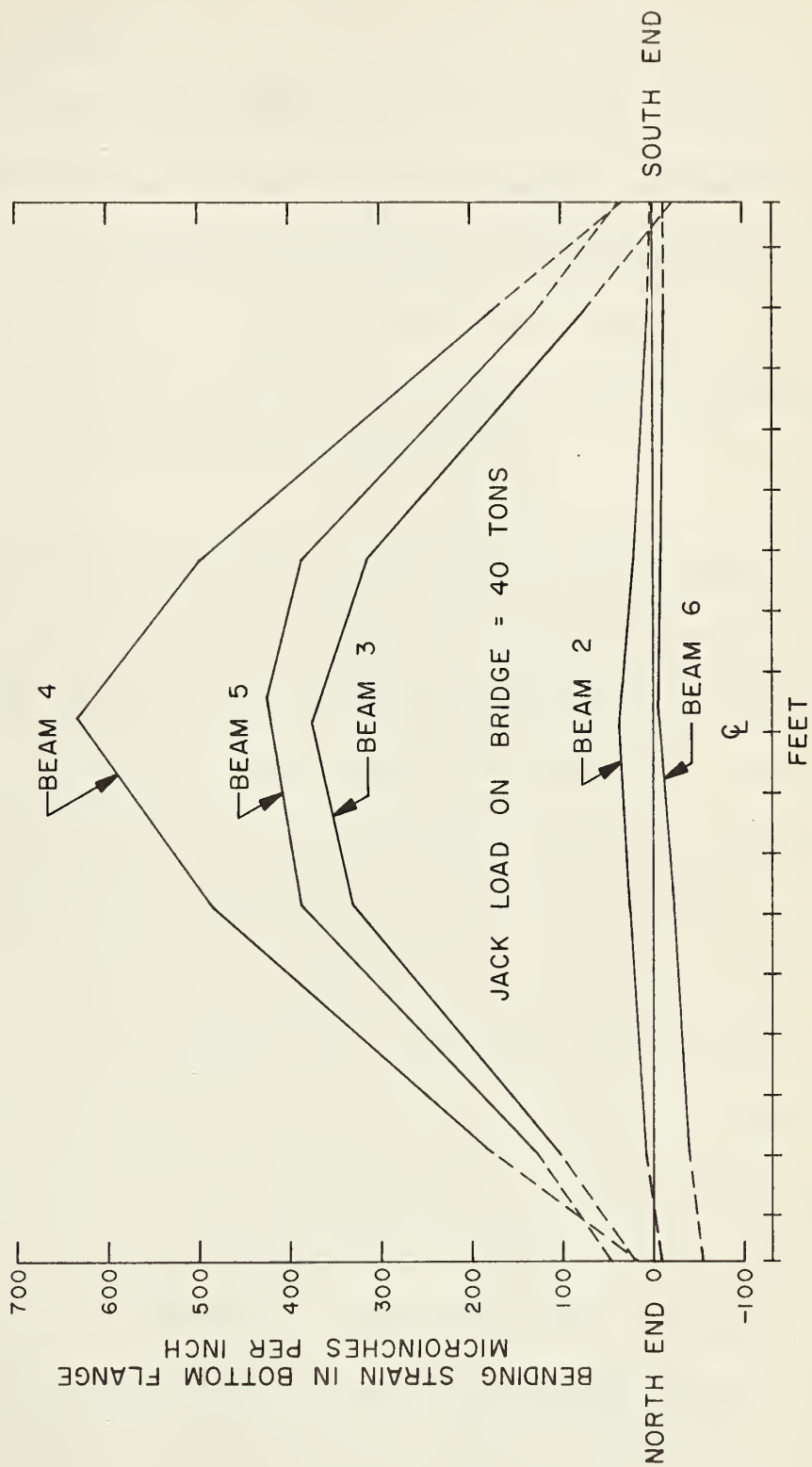


FIGURE 4.19 - TYPICAL FLOOR BEAM BENDING STRAIN DIAGRAMS - OCTOBER TEST

the moment diagrams strongly suggested that the end moments were indeed slightly incorrect.

Curves were nevertheless successfully fitted to the data as presented to the computer. The resulting shearing force and loading diagrams were, however, entirely unsatisfactory in terms of the known total loads, confirming the observations made in the foregoing paragraph. So the determining of the exact magnitude and distribution of load on a floor beam was found to be impossible.

Distribution of load on a beam as related to the Slope-Deflection

Analysis. It will be recalled that the solution of the slope-deflection equations of Chapter II requires a knowledge of the fixed-end moments which would be produced in the floor beams by the actual loads. In order to compare directly the analysis and the test results, it became necessary to discover some load distribution which would produce bending moment diagrams approaching those found in the tests. An attempt was made for the October data to break down the measured bending moment curves into component bending moments due to end moments, two symmetrically placed but unequal concentrated loads, a uniformly distributed load, and a parabolically distributed load. There were thus established six unknowns. The actual moment of any point on a beam could then be equated to the sum of the moments at that point due to the chosen loading components. Since moments were known at five points, five equations could be set up. The necessary

sixth equation expressed the fact that the sum of the load components was equal to the total load. A set of equations was solved for each load increment and for each loaded beam in the October test. In general, the results indicated that the bending moment diagrams were most strongly influenced by the parabolic component of the load, although the proportions varied. The actual numerical values cannot be considered significant.

The bending moment curves produced by both uniform and parabolic loads were then superimposed on the experimental moment diagrams in order to make a visual comparison. The curves due to the parabolic loading were seen to more closely approximate the shape of the experimental diagrams, making allowance for vertical displacement due to the unknown end moments.

A simple theoretical investigation disclosed that, where the total load is the same regardless of distribution, the fixed-end moment due to a parabolic load falls almost midway between those due to a uniform load and two equal concentrated loads positioned as in the tests. Since the actual load distribution on a beam was undoubtedly between these limits, it was decided that using a parabolic load distribution to compute fixed-end moments would probably be the most satisfactory method.

Although these remarks have been concerned primarily with the October test, because of the suitability of that data, it was felt

nonetheless that the load distribution in the July test would not differ materially, and a parabolic distribution was again used.

Distribution of load among the floor beams. Figures 4.20 and 4.21 illustrate load-moment relationships for corresponding points on all floor beams. That is to say, each diagram comprises bending strains measured at the same relative position on every floor beam. Since the measured bending moment curves are similar in shape for all floor beams, it is possible to gain some impression of the relative magnitude of the load carried by each beam.

Tentatively accepting the curves as indicating moment due only to external load, it is obvious that at no time does any theoretically unloaded beam carry significantly more than about ten percent of the load applied to the most heavily loaded beam. In the case of the October test, Figure 4.21, there is fairly satisfactory evidence that the beam adjacent to the center floor beam carried, as planned, loads of approximately 70.7 percent of the magnitude of the load on the center beam.

These curves, however, include not only the effects of external load, but also the effects of end moment on the floor beams. If the moments in the verticals are examined closely, it will be found that in virtually every case the direction of moment induced by the vertical at the end of the floor beam would tend to produce the deviations from "theoretical" in the directions seen in Figures 4.20 and 4.21.

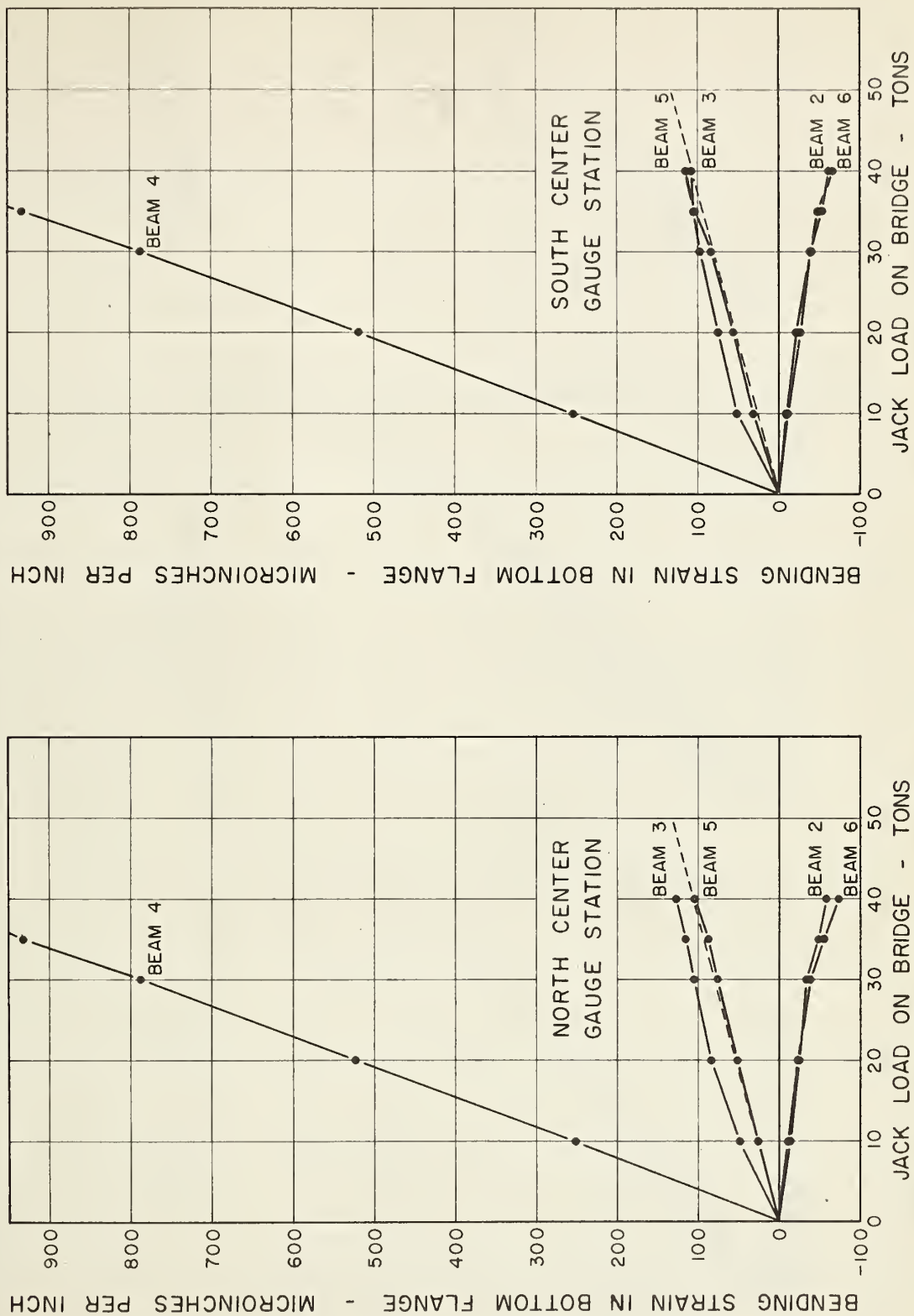


FIGURE 4.20 - FLOOR BEAMS: BENDING STRAINS VERSUS LOAD - JULY TEST

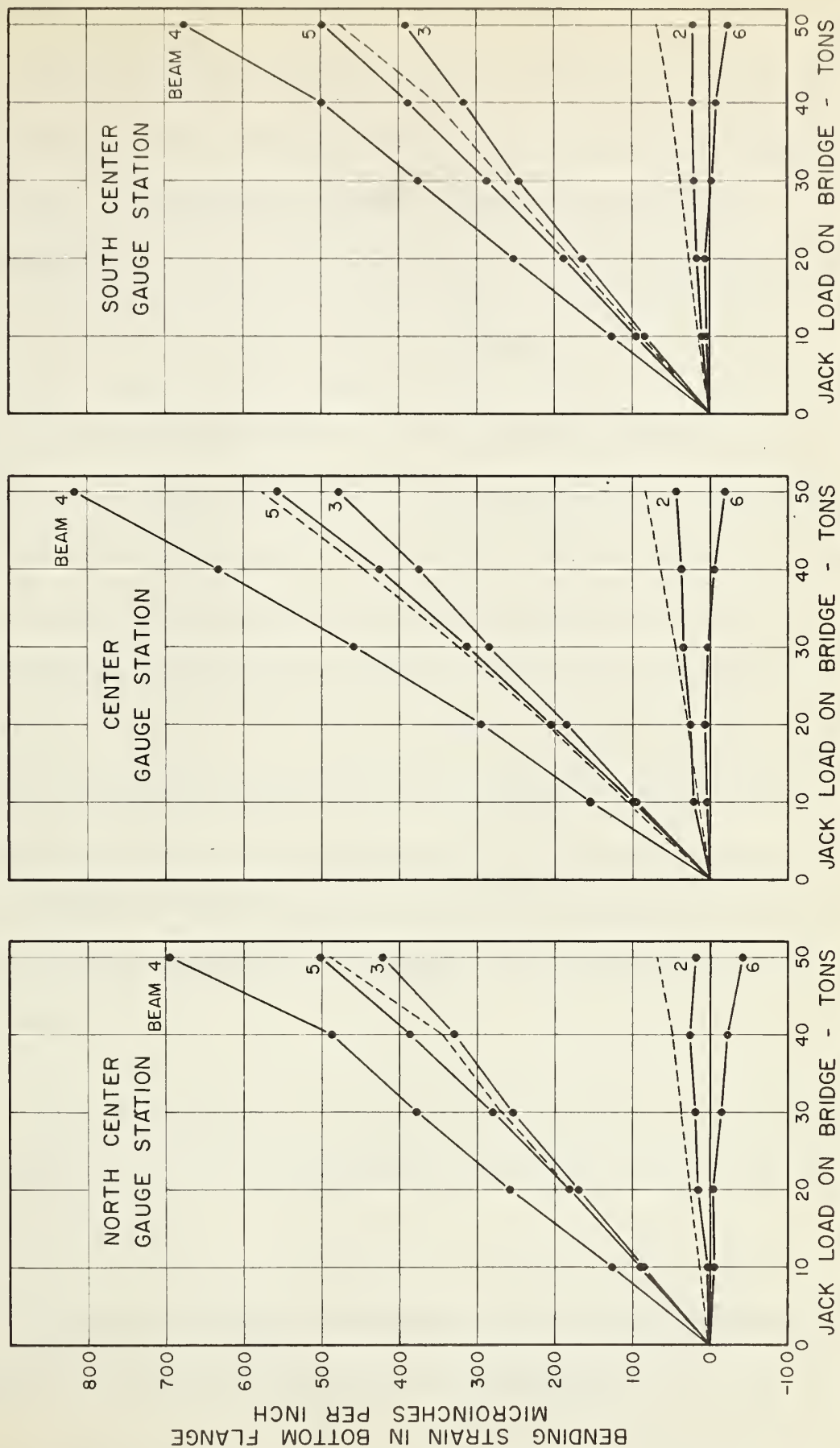


FIGURE 4.21 - FLOOR BEAMS: BENDING STRAINS VERSUS LOAD - OCTOBER TEST

It is not suggested that the only source of discrepancy in the measured floor beam bending moments is the action of the vertical members. It is suggested, however, that any unaccounted for discrepancies are of such magnitude as to be of little significance.

IV. LOAD TRANSFER

The term "load transfer" has been used in reference to the phenomenon whereby load applied to one floor beam may be partially distributed, or transferred, to adjacent or other floor beams, primarily, it is supposed, through the action of the floor deck. Some controversy exists concerning whether or not, and to what degree, load transfer occurs in a bridge of this type. Despite the absence of conclusive supporting evidence, it is the opinion of the author that, while some transfer of load does take place, the amount is negligible. This conclusion has been reached on the basis of information contained in the preceding sections of this chapter, and to which reference is made below.

Beam deflections. The centerline deflections of the floor beams as compared with their end deflections would indicate very little, if any, load transfer.

Diagonal members. The available data for the diagonals tends to confirm the absence of appreciable load transfer.

Vertical members. The moment-deflection relationships for the verticals further suggest that only nominal load transfer occurs.

Floor beams. Whereas data concerning the floor beams should have definitively answered the load transfer question, circumstances previously described in detail have prevented more than another qualitative evaluation. Even this, however, agrees with other indications.

For these reasons, then, the author feels justified in stating that the amount of load transferred from one beam to another, by the bridge deck or other means, is very small and could be ignored in analysis.

V. COLLAPSE OF THE BRIDGE

In both of the 1960 tests the actual collapse of the bridge was accompanied by the yielding of the top chord of the south truss. The distribution of load on the bridge, however, played an important part in determining the manner in which failure was initiated. An attempt has been made to explain both failures with reference to the buckling analysis presented in Chapter II.

At this point, it might be well to emphasize that the aforementioned analysis is an analysis of only the stability of the top chord of the truss. Buckling of the top chord does not necessarily imply

collapse of either the truss, or of the bridge as a whole. Unless otherwise specified, collapse shall be taken to mean collapse of the bridge, irrespective of the immediate cause.

The analysis requires that the state of stress in each member be known relative to the yield stress, and relative to local elastic instability. The value of the modulus of elasticity is an integral part of the stiffness of a member, and in order to accurately evaluate the collapse load, the proper modulus, whether the modulus of elasticity, or the tangent modulus, must be employed. The use of such a refinement as the tangent modulus, however, calls for a detailed knowledge of the stress-strain curve; at best its value is a matter of controversy. For this reason, the computer program was set up to accept only one value of the modulus of elasticity for use throughout the bridge. Any member whose stress exceeded the yield stress was considered to have a stiffness equal to zero, and the pertinent input data could be altered to produce this effect in the analysis.

The buckling load as calculated by the previously described method of determinants therefore represents, in general, the load which could be attained if neither yielding nor local buckling of any member occurred. Theoretically, buckling of the top chord would occur unaccompanied by any yielding.

In actuality, it is unlikely that so high a load could be reached, since secondary stresses would produce yielding at a much lower load,

thus changing the state of equilibrium of the top chord. Such indeed was the case with the present tests. In each, the true collapse load was far less than the predicted elastic buckling load.

The July test. The collapse load found in the July test was 45 tons. A single plastic hinge formed in the top chord of the south truss shortly after the failure load was reached. Figure 4.22 shows a general view of the bridge following the test. The south truss is on the left, and the plastic hinge is readily visible. It might be observed that the darker-colored members in the photograph were those which were purchased to replace the members damaged in the 1959 test. The south chord is shown more clearly in Figure 4.23. Figure 4.24 illustrates the behavior of the member at the plastic hinge. Only one photograph is available which was taken during the test. Repeated flash failure has resulted in multiple exposure of the light bulbs and numerous bright reflections. Despite these shortcomings, Figure 4.25 shows fairly clearly the shape of the south chord prior to collapse.

As has been pointed out, the loading conditions of the July test were such that the entire load was applied to the center floor beam. Disregarding possible yielding of any bridge members, determinants were computed for loads ranging from 38 to 50 tons. These are shown graphically in Figure 4.26(a). The predicted buckling load obviously exceeds 50 tons.

It is known from the 1959 test, however, that the center floor

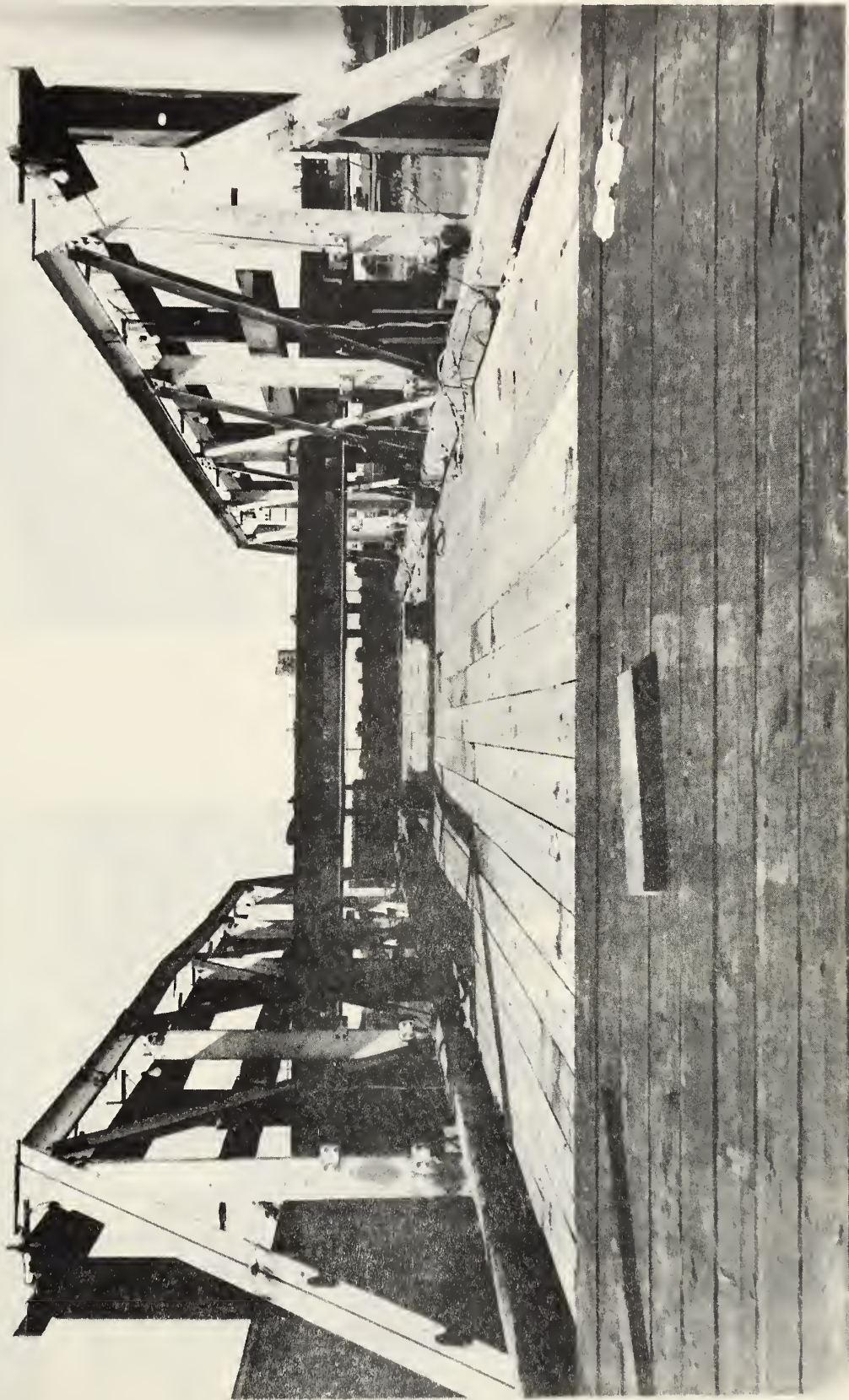


FIGURE 4.22 - TEST BRIDGE - JULY TEST (LOOKING WEST)

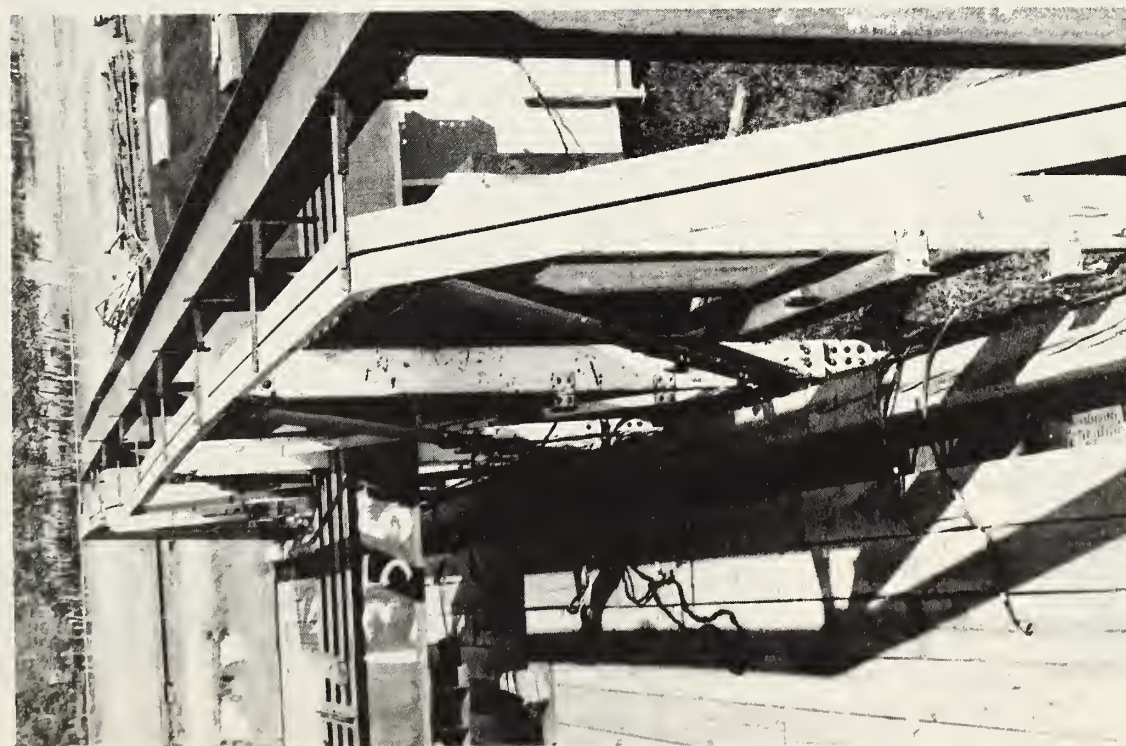


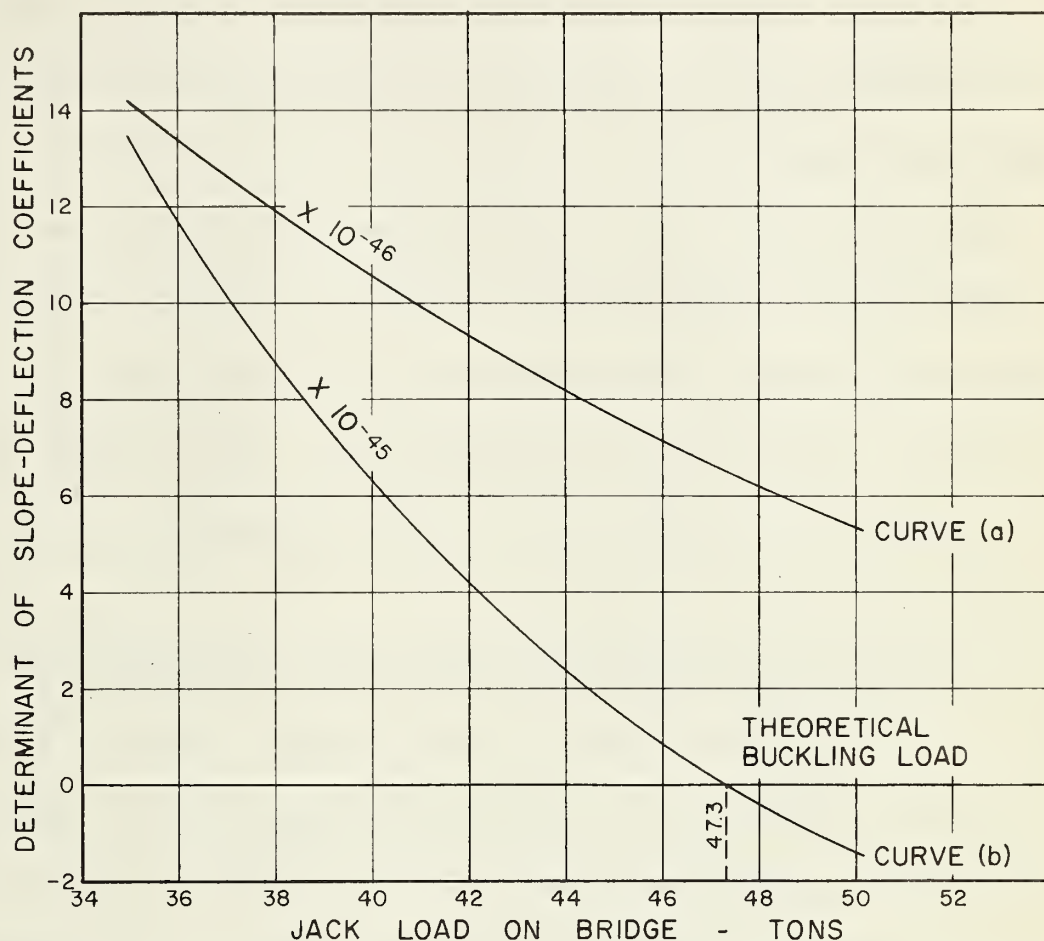
FIGURE 4.23 - SOUTH TRUSS - JULY TEST



FIGURE 4.24 - PLASTIC HINGE
IN SOUTH CHORD - JULY TEST



FIGURE 4.25 - SOUTH TRUSS DURING JULY TEST



CURVE (a): NO LOCAL BUCKLING NOR YIELDING THROUGHOUT BRIDGE
 CURVE (b): PLASTIC HINGE IN CENTER FLOOR BEAM

FIGURE 4.26 - DETERMINANTS OF COEFFICIENTS OF SLOPE-DEFLECTION EQUATIONS - JULY TEST

beam yielded at a load somewhere in the range of from 25 to 30 tons, and that a so-called plastic hinge had certainly formed (see Figure 3.4) by the time the load had passed 40 tons. This phenomenon effectively reduced the stiffness of the floor beam to zero. Because the failure load of the July test exceeded that of the 1959 test by some three tons, the strains in the floor beam had passed the strain-hardened proportional limit, and a plastic hinge again existed. The computer input data was therefore adjusted to set equal to zero the stiffness of the center floor beam. Determinants were recomputed, this time with the results shown in Figure 4.26(b). The buckling load thus predicted is seen to be 47.3 tons, comparing favorably with the experimental value of 45 tons.

It is important to realize that the buckling of, or even the formation of a plastic hinge in, the top chord would in itself be insufficient to cause collapse of the truss. A consideration of statics (Figure 4.27) shows that due to the statical indeterminacy of the truss, the yielded member could actually be removed without causing instability. In order for the truss to collapse, one of the diagonal members would have to be incapacitated. Probably the diagonal initially in compression would buckle, especially since the force in the tension diagonal would be reduced. No evidence of this was found, however (see Figure 4.22), and it is therefore supposed that the truss itself did not collapse.

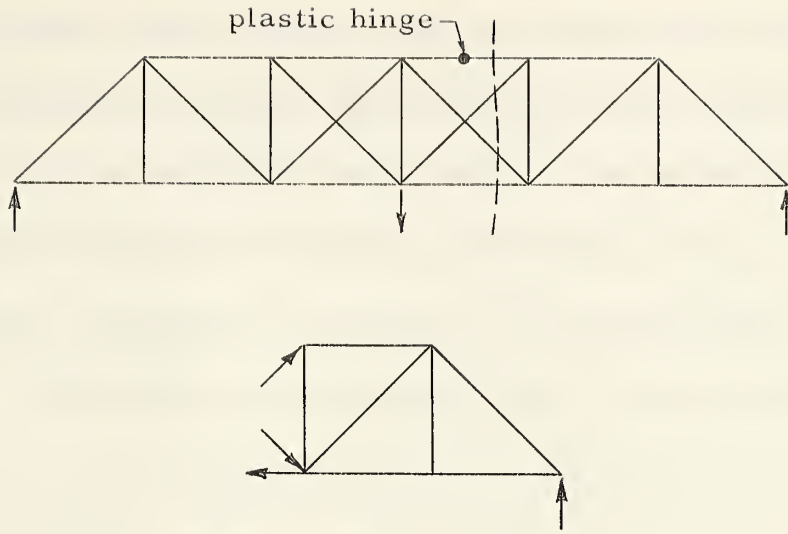


Figure 4.27

The collapse of the bridge may be explained as follows, with reference to Figure 4.28:

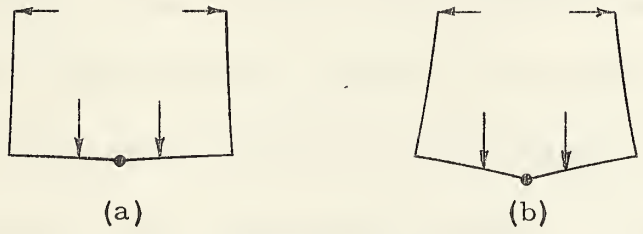


Figure 4.28

Figure 4.28(a) represents the equilibrium condition of the center floor beam just prior to buckling of the top chord. The outward forces at the top of the verticals produce end moments on the floor beam, preventing excessive deflection of the latter. Elastic buckling of the top chord, however, permits the top of the vertical to move inward without any

increase in the lateral force. Thus any attempted increase in the load on the floor beam merely increases the lateral deflection of the vertical, and hence the vertical deflection of the beam, since the top chord can sustain no additional lateral force. This explains why, during the test, the pistons of the jacks continued to travel, while the pressure gauges registered no increase, for an appreciable interval of time prior to the formation of the plastic hinge in the top chord. Obviously, then, the buckling load of the top chord is only one of the variables of which the collapse load of the truss is a function.

Unless the floor beam were known to have yielded, accurate evaluation of the upper chord buckling load would clearly have been impossible. Of all the members of the bridge, the floor beams are probably the easiest to analyze approximately, since the only manifestation of secondary stress is in the form of end moments induced by the verticals. In a case where the stiffness of the beams is large relative to that of the vertical members, it is unlikely that a serious error would be introduced in analyzing the beams as simply supported, in order to establish the yield load. It should be borne in mind, though, that such an error could be on the unsafe side (see Figure 4.14). A more accurate analysis would require the computation of the secondary stresses.

The collapse load as determined by the yield strength of the floor beams could be interpreted as an ultimate axle load. It is

suggested that by applying suitable safety factors to this ultimate load, a maximum permissible axle load could be established, as distinct from a maximum permissible total load on the bridge. Cognizance should be taken of the fact that the tests were static in nature, whereas a vehicle represents a dynamic loading.

The October test. Distribution of the load among three beams in the October test raised the collapse load of the bridge to 65 tons. This time, two plastic hinges formed antisymmetrically in the south chord. A general view of the bridge after the test is shown in Figure 4.29. The south truss is seen on the right; the two plastic hinges are clearly visible. Figures 4.30 and 4.31 show the deformed chord to better advantage. The eastward plastic hinge is seen in Figure 4.32. Figures 4.33 and 4.34 show the buckled condition of the compression diagonals of the south truss.

According to strain measurements, none of the floor beams had yielded prior to collapse. Determinants were therefore calculated on the basis that all members were stressed below the yield stress, and furthermore, that no local buckling of individual members had occurred. These determinants are shown graphically in Figure 4.35(a). As in the July test, the buckling load thus determined is substantially in excess of the collapse load.

Approaching the problem in a manner similar to that used for the July test, the determinants were recalculated with the stiffnesses

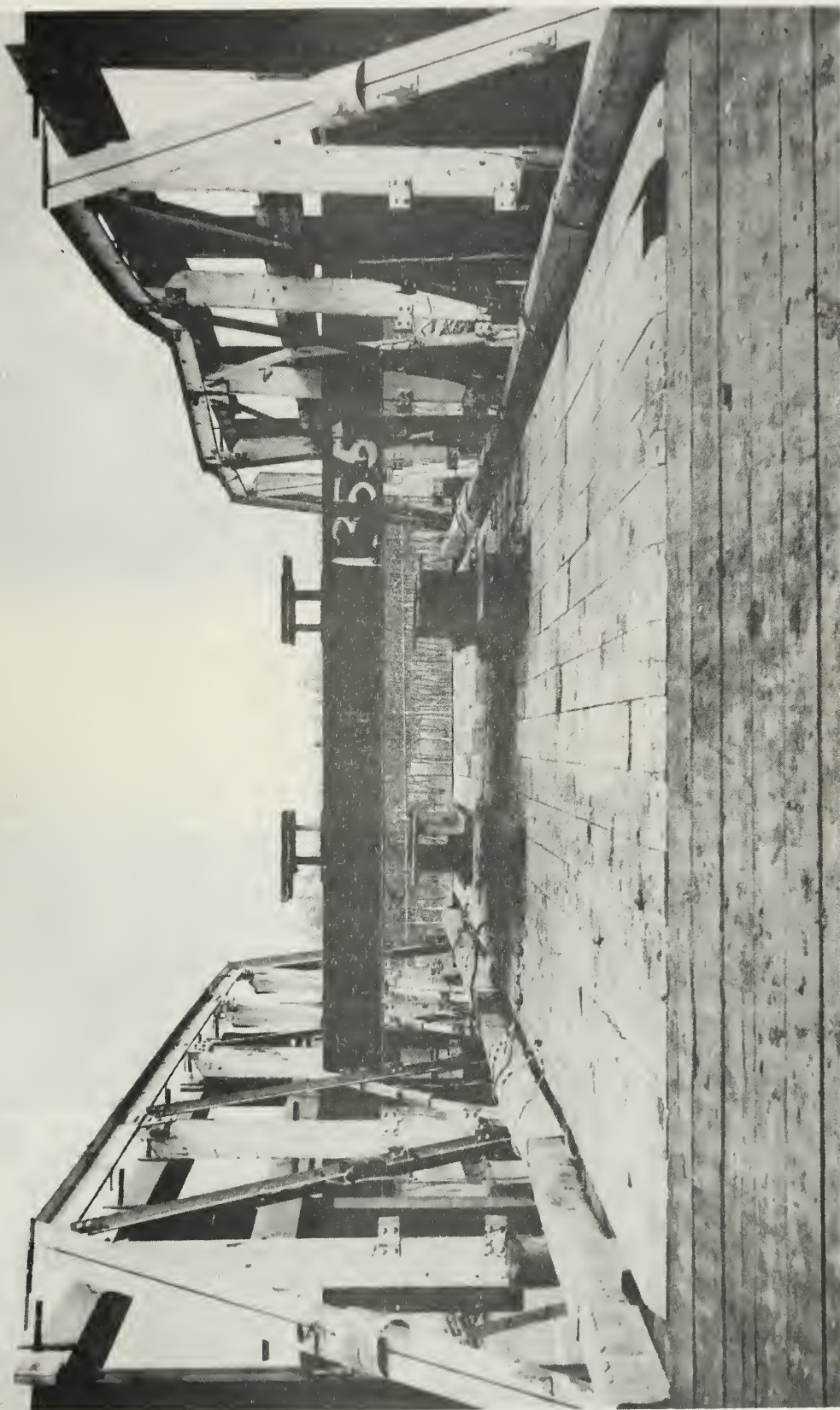


FIGURE 4.29 - TEST BRIDGE - OCTOBER TEST (LOOKING EAST)

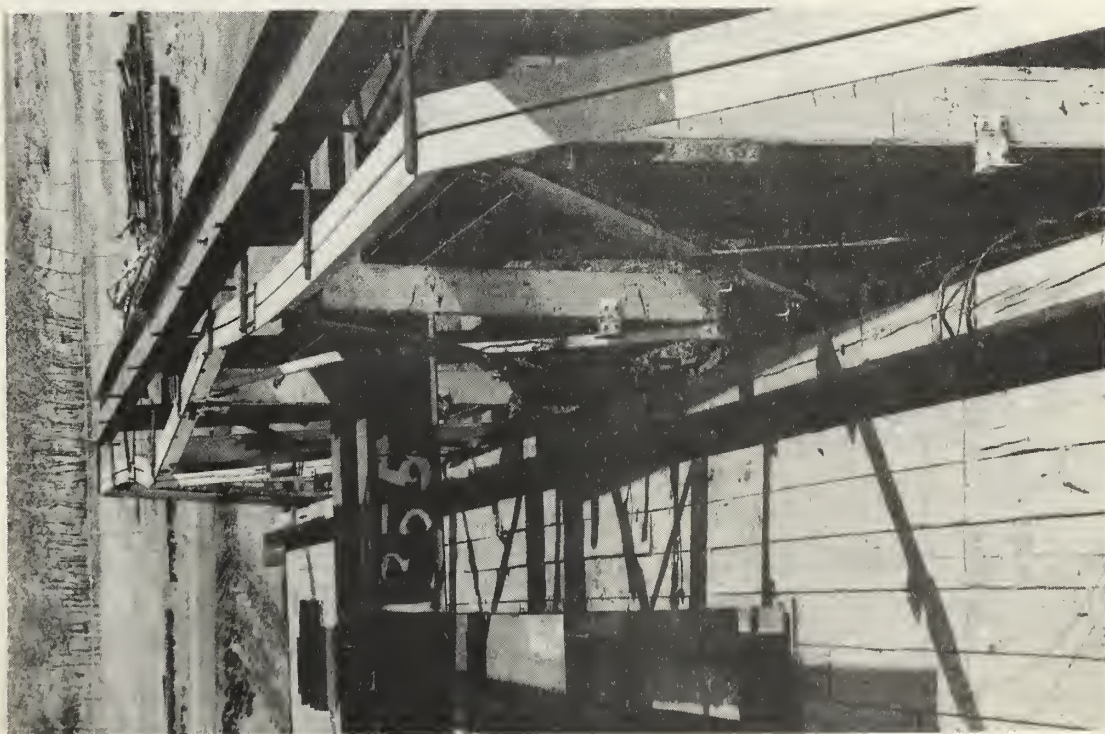


FIGURE 4.30 - SOUTH TRUSS
OCTOBER TEST



FIGURE 4.31 - SOUTH CHORD
OCTOBER TEST



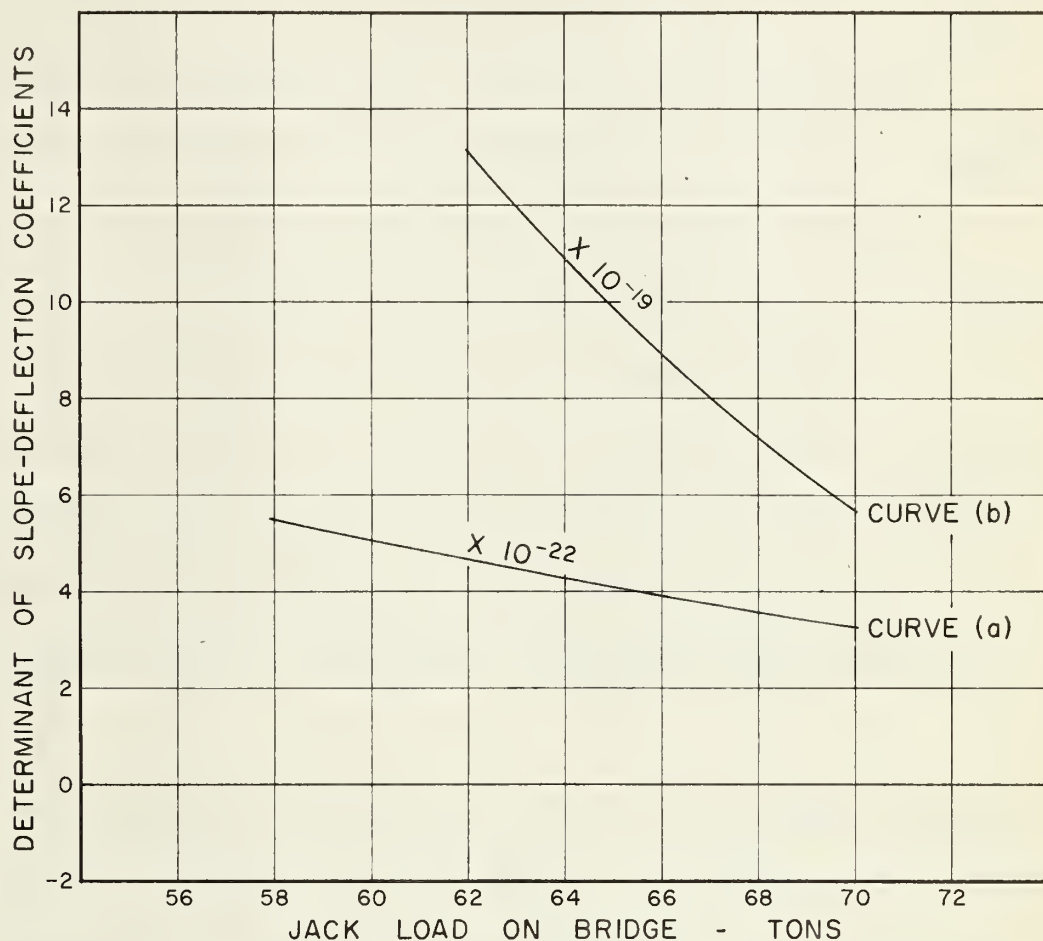
FIGURE 4.32 - EASTERLY PLASTIC HINGE
OCTOBER TEST



FIGURE 4.33 - COMPRESSION DIAGONALS
SOUTH TRUSS - OCTOBER TEST
(LOOKING WEST)



FIGURE 4.34 - COMPRESSION DIAGONALS, SOUTH TRUSS
OCTOBER TEST (LOOKING EAST)



CURVE (a): NO LOCAL BUCKLING NOR YIELDING THROUGHOUT BRIDGE

CURVE (b): PLASTIC HINGES IN CENTER PANELS OF TOP CHORD

NOTE: THESE CURVES HAVE BEEN COMPUTED FOR ONE-HALF OF TRUSS, ASSUMING ANTISYMMETRICAL BUCKLING

FIGURE 4.35 - DETERMINANTS OF COEFFICIENTS OF SLOPE-DEFLECTION EQUATIONS - OCTOBER TEST

of the yielded top chord members equal to zero. These determinants are shown in Figure 4.35(b). Again, the buckling load is seen to be in excess of the collapse load of the bridge. This would indicate that the top chord did not buckle, but rather that secondary stresses caused yielding below the buckling load. Further justification for this viewpoint is discovered when the test behavior is considered. Whereas in the July test, excessive vertical deflection took place without increase of load immediately before the formation of the plastic hinge, very little such deflection occurred in the October test prior to yielding of the top chord. As a result, the plastic hinges formed with comparatively little warning. With this particular load distribution, then, the top chord buckling load does not represent the lowest collapse load, and an investigation of secondary stresses would be necessary.

In explanation of the ultimate collapse of the bridge, consider Figure 4.36: Applying the laws of statics to the section of the truss, it is apparent that complete removal of both yielded members would not produce instability. It is obvious that under these circumstances, the original compression diagonals would suffer a severe increase in stress. In fact, these diagonals were known to have buckled in the October test (see Figures 4.33 and 4.34). Failure of these members, of course, would now cause collapse of the truss, and collapse of either truss implies collapse of the bridge.

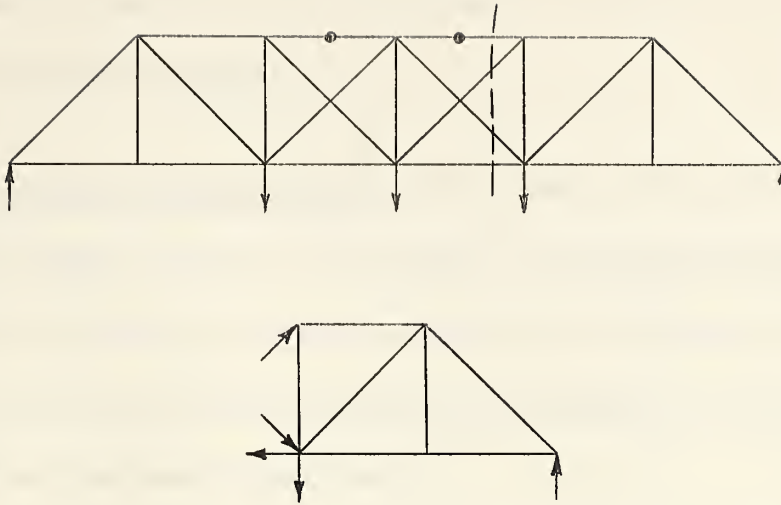


Figure 4.36

It is felt that the foregoing is a valid explanation of the October failure of the bridge. It has once more been demonstrated that buckling of the top chord is not the sole criterion of collapse.

VI. TOP CHORD DISPLACEMENTS

The successful evaluation of secondary stresses in a pony-truss bridge depends heavily upon the accuracy with which the displacements of the top chord can be calculated. The only such displacements recorded during the tests were the lateral deflections of the top chord panel points. The rotations about the longitudinal and vertical axes would be very difficult, if not impossible, to measure. Thus, of the computed displacements, only the lateral deflections have been presented in detail. While the correspondence of analytical and experimental results is highly encouraging, deficiencies in both the

analysis and the test data have precluded the possibility of computing accurate secondary stresses.

Theoretical displacements. The computed displacements are recorded in Tables 4.3, 4.4, 4.5, and 4.6, and the theoretical lateral deflections are illustrated by the broken lines in Figures 4.37 through 4.42. The latter have been obtained by subtracting the initial deflections from the computed deflections.

Computation of the top chord displacements as set forth by Lee and Clough requires a knowledge of the initial lateral deflections of the top chord, where the top chord is taken to include the end diagonals. That is, these deflections must be known relative to the ends of the chord, the reaction points. The initial lateral deflections of the test bridge, however, were measured relative to the first interior panel points, whose own initial deflections were not measured. This deficiency was impossible to overcome during analysis of the test results, although some estimate was possible because chord-to-chord distances had been measured at the panel points. The difference between these distances and the distance between the reactions, was divided equally between the corresponding first interior panel points, and all other initial deflections were corrected accordingly. At best, this procedure must be considered a makeshift expedient.

Shortcomings in the analysis have also appeared. No provision has been made for the separation of the effects of live and dead load,

TABLE 4.3

THEORETICAL TOP CHORD JOINT DISPLACEMENTS
NORTH TRUSS, JULY TEST

LOAD	JOINT	δ	α	β	
TONS		INCHES	RADIANS X 1000	RADIANS X 1000	
10	2	-0.100	0.449	0.317	
	3	0.536	-1.067	1.588	
	4	0.753	-2.215	0.007	
	5	0.283	-1.027	-1.677	
	6	-0.724	0.717	-0.426	
20	2	-0.157	1.074	0.607	
	3	0.632	-2.134	3.485	
	4	1.107	-4.875	0.023	
	5	0.376	-2.065	-3.673	
	6	-0.807	1.622	-0.836	
30	2	-0.234	1.919	0.840	
	3	0.726	-3.188	5.795	
	4	1.514	-8.122	0.053	
	5	0.469	-3.109	-6.096	
	6	-0.911	2.764	-1.204	
35	2	-0.282	2.443	0.920	
	3	0.772	-3.705	7.152	
	4	1.743	-10.031	0.077	
	5	0.515	-3.630	-7.516	
	6	-0.973	3.444	-1.359	
40	2	-0.338	3.047	0.963	
	3	0.816	-4.212	8.680	
	4	1.993	-12.183	0.108	
	5	0.560	-4.149	-9.114	
	6	-1.044	4.211	-1.484	
INITIAL DEFLECTIONS, INCHES					
JOINT	2	3	4	5	6
DEFLECTION	-0.06	0.44	0.44	0.19	-0.66

TABLE 4.4

THEORETICAL TOP CHORD JOINT DISPLACEMENTS
SOUTH TRUSS, JULY TEST

LOAD	JOINT	δ	α	β	
TONS		INCHES	RADIANS X 1000	RADIANS X 1000	
10	2	-0.084	0.263	-0.053	
	3	-0.306	-0.602	1.573	
	4	0.815	-2.345	0.357	
	5	0.555	-1.274	-1.824	
	6	-0.731	0.797	-0.562	
20	2	-0.122	0.671	-0.204	
	3	-0.261	-1.117	3.442	
	4	1.183	-5.164	0.823	
	5	0.675	-2.612	-4.019	
	6	-0.823	1.809	-1.136	
30	2	-0.177	1.255	-0.511	
	3	-0.228	-1.499	5.700	
	4	1.608	-8.608	1.453	
	5	0.802	-4.034	-6.719	
	6	-0.940	3.095	-1.708	
35	2	-0.211	1.627	-0.749	
	3	-0.218	-1.619	7.017	
	4	1.848	-10.636	1.858	
	5	0.869	-4.787	-8.321	
	6	-1.011	3.867	-1.985	
40	2	-0.252	2.061	-1.066	
	3	-0.215	-1.674	8.488	
	4	2.111	-12.924	2.352	
	5	0.939	-5.576	-10.143	
	6	-1.091	4.745	-2.251	
INITIAL DEFLECTIONS, INCHES					
JOINT DEFLECTION	2	3	4	5	6
	-0.06	-0.36	0.49	0.44	-0.66

TABLE 4.5

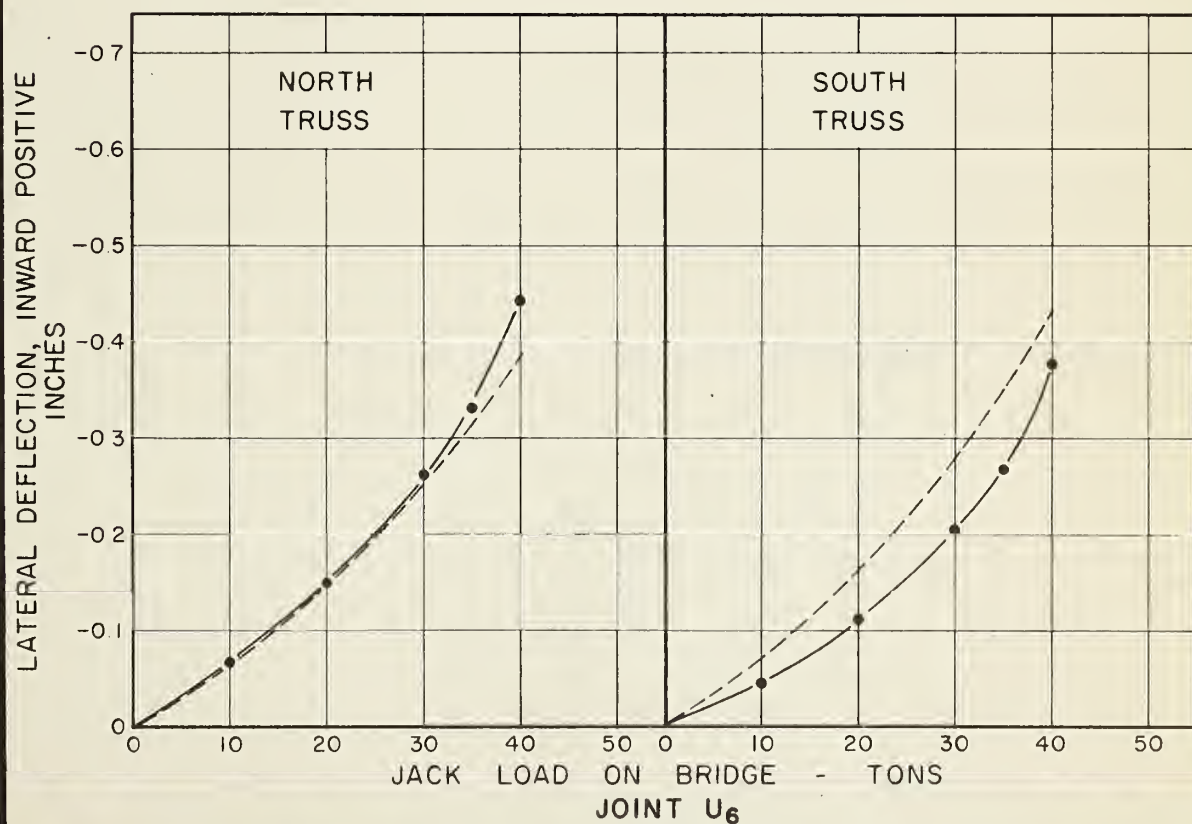
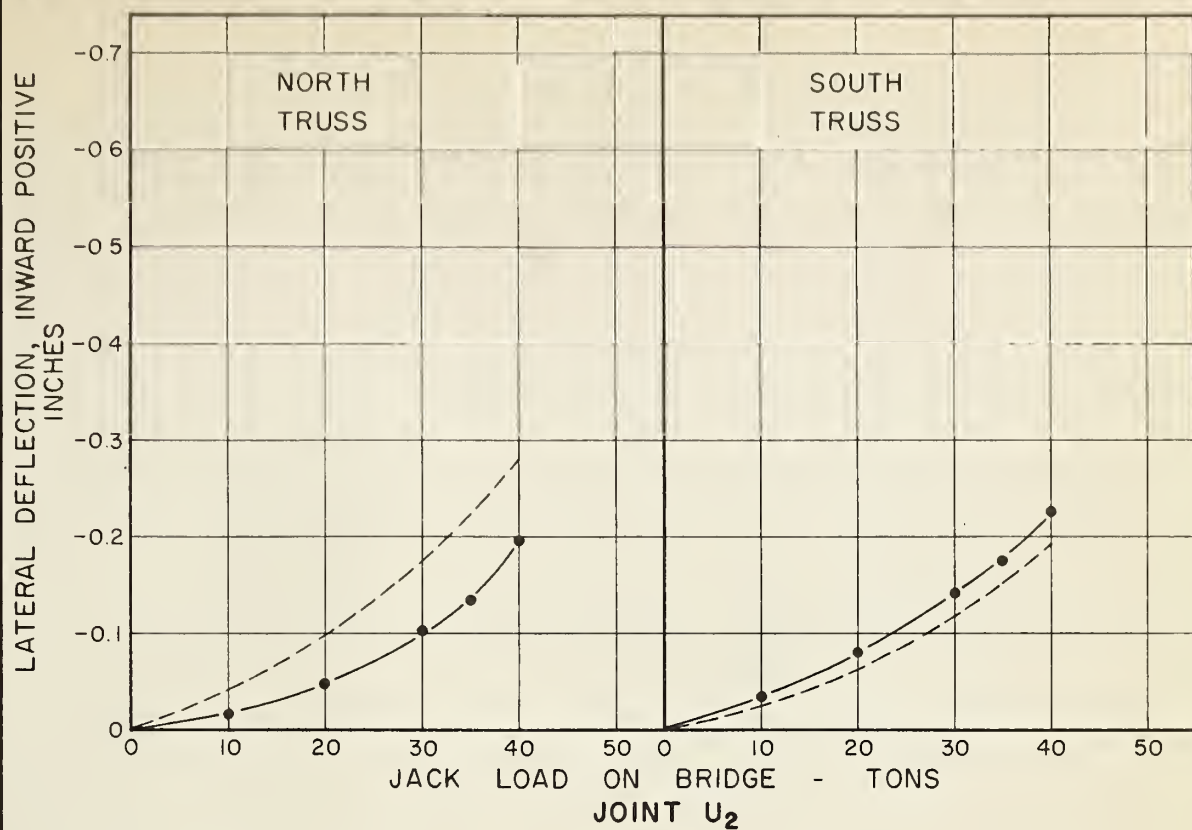
THEORETICAL TOP CHORD JOINT DISPLACEMENTS
NORTH TRUSS, OCTOBER TEST

LOAD	JOINT	δ	α	β	
TONS		INCHES	RADIANS X 1000	RADIANS X 1000	
10	2	-0.064	0.074	0.743	
	3	0.569	-1.351	1.000	
	4	0.594	-2.066	0.078	
	5	0.539	-1.455	-1.123	
	6	-0.693	0.394	-0.954	
20	2	-0.085	0.312	1.376	
	3	0.696	-2.388	1.856	
	4	0.793	-3.749	0.183	
	5	0.678	-2.634	-2.121	
	6	-0.744	0.976	-1.829	
30	2	-0.118	0.686	2.075	
	3	0.827	-3.474	2.839	
	4	1.010	-5.619	0.325	
	5	0.827	-3.913	-3.277	
	6	-0.812	1.727	-2.810	
40	2	-0.167	1.227	2.849	
	3	0.964	-4.620	3.994	
	4	1.249	-7.740	0.522	
	5	0.987	-5.324	-4.647	
	6	-0.899	2.688	-3.924	
50	2	-0.235	1.966	3.702	
	3	1.105	-5.817	5.344	
	4	1.515	-10.155	0.803	
	5	1.162	-6.894	-6.280	
	6	-1.011	3.908	-5.197	
INITIAL DEFLECTIONS, INCHES					
JOINT	2	3	4	5	6
DEFLECTION	-0.06	0.40	0.34	0.36	-0.66

TABLE 4.6

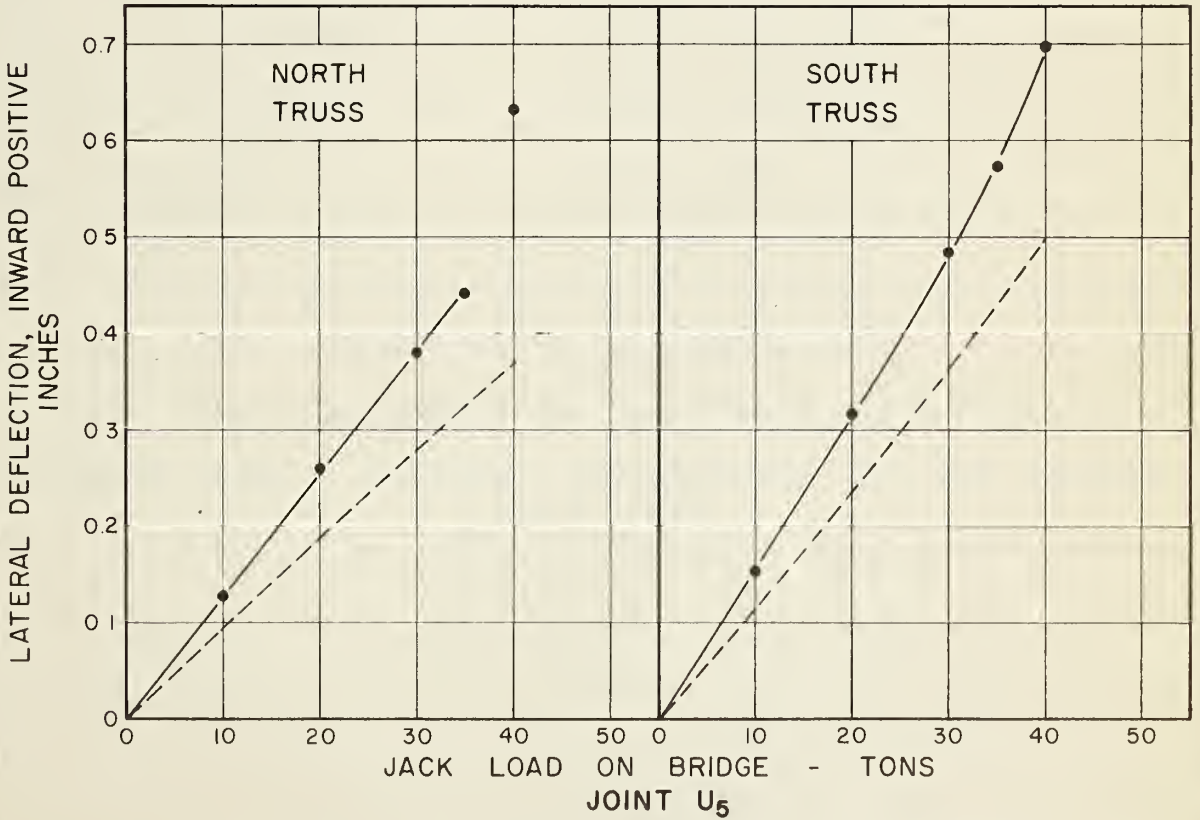
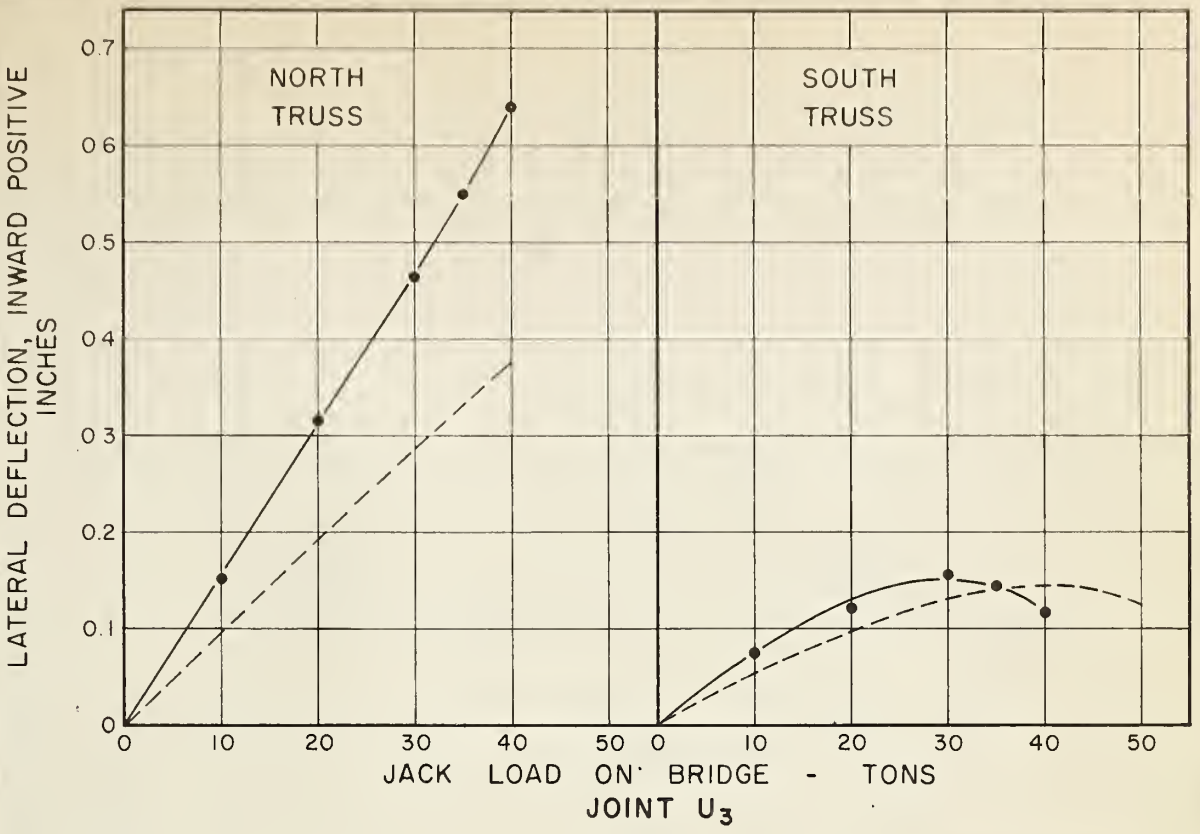
THEORETICAL TOP CHORD JOINT DISPLACEMENTS
SOUTH TRUSS, OCTOBER TEST

LOAD	JOINT	δ	α	β	
TONS		INCHES	RADIANS X 1000	RADIANS X 1000	
10	2	-0.036	-0.255	0.328	
	3	-0.599	-0.929	0.932	
	4	0.145	-2.076	0.253	
	5	-0.038	-1.384	-1.140	
	6	-0.677	0.213	-0.797	
20	2	-0.023	-0.385	0.479	
	3	-0.517	-1.480	1.712	
	4	0.346	-3.781	0.575	
	5	0.095	-2.489	-2.179	
	6	-0.711	0.605	-1.496	
30	2	-0.019	-0.428	0.610	
	3	-0.437	-1.995	2.613	
	4	0.566	-5.689	0.995	
	5	0.237	-3.696	-3.408	
	6	-0.761	1.157	-2.279	
40	2	-0.025	-0.365	0.699	
	3	-0.363	-2.460	3.675	
	4	0.811	-7.874	1.556	
	5	0.392	-5.043	-4.905	
	6	-0.829	1.913	-3.167	
50	2	-0.042	-0.187	0.706	
	3	-0.298	-2.822	4.919	
	4	1.087	-10.392	2.335	
	5	0.562	-6.569	-6.750	
	6	-0.923	2.928	-4.182	
INITIAL DEFLECTIONS, INCHES					
JOINT DEFLECTION	2 -0.06	3 -0.73	4 -0.11	5 -0.21	6 -0.66



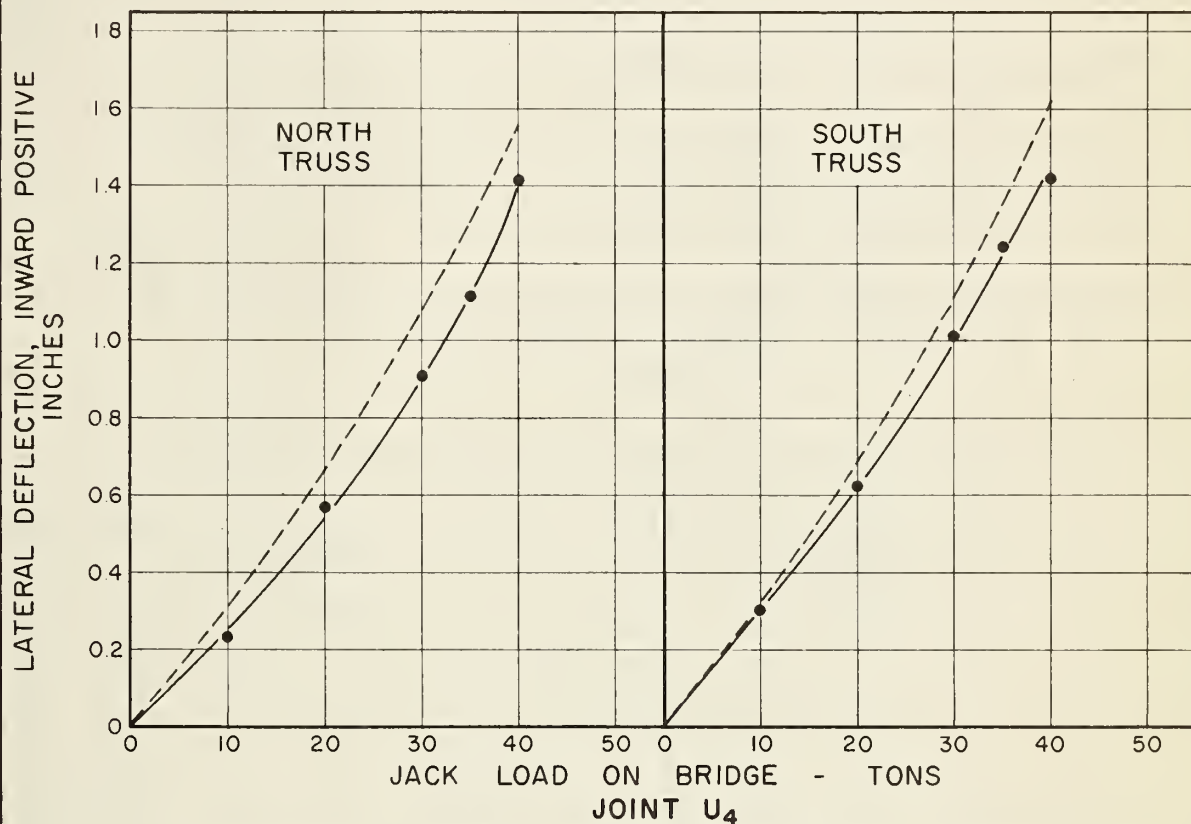
THEORETICAL DEFLECTIONS ILLUSTRATED BY BROKEN LINES

FIGURE 4.37 - LATERAL DEFLECTIONS OF TOP CHORDS
JULY TEST



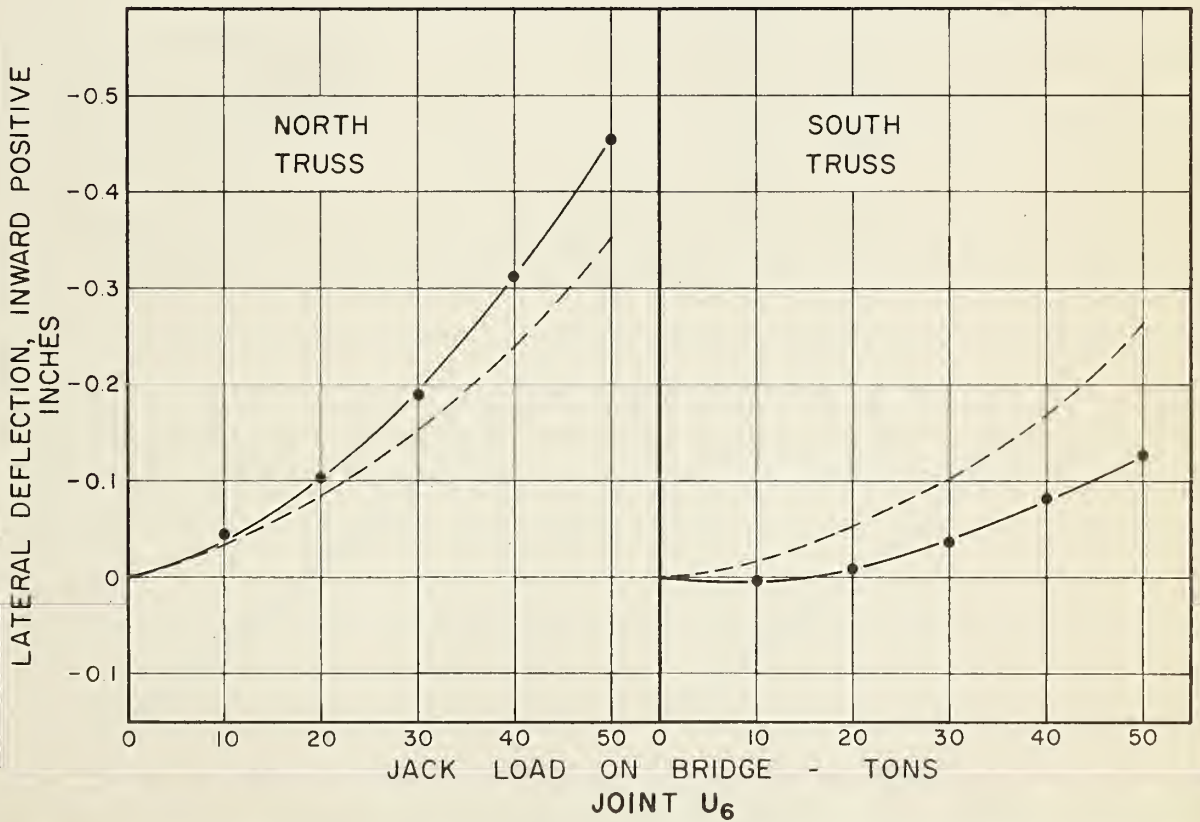
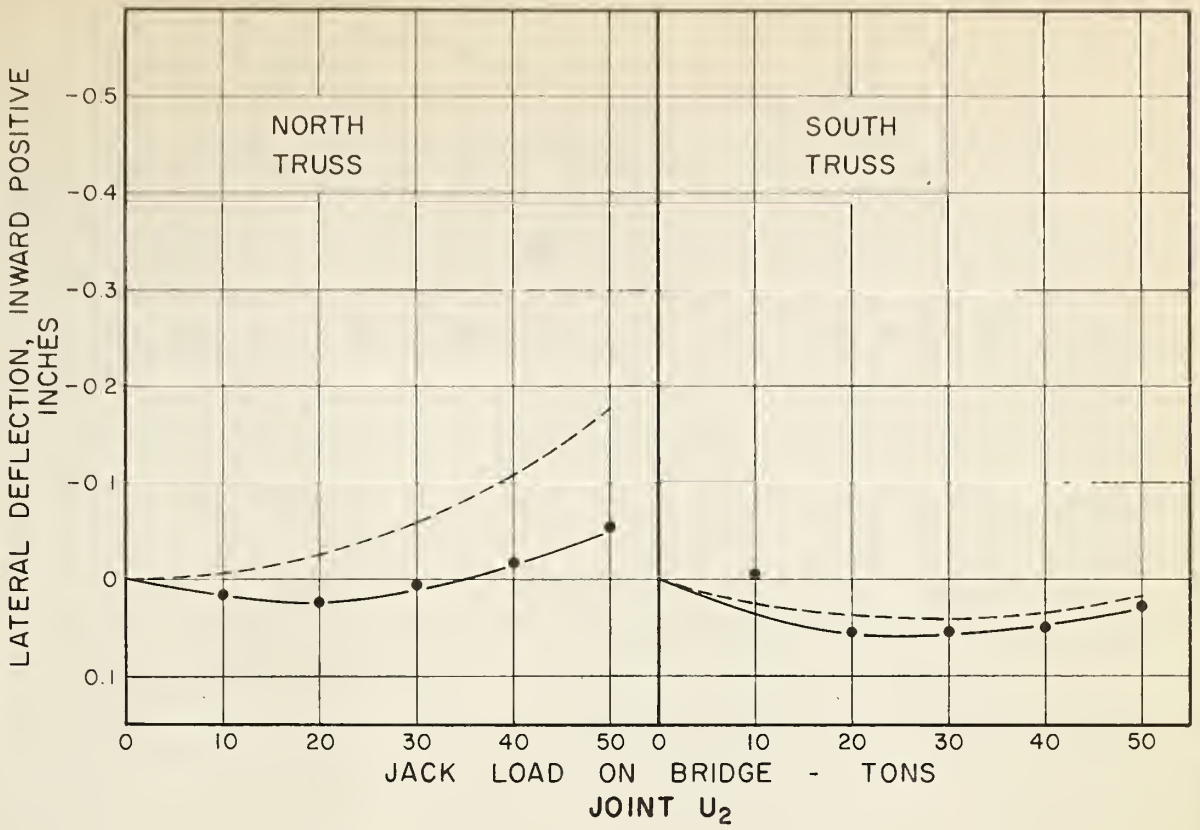
THEORETICAL DEFLECTIONS ILLUSTRATED BY BROKEN LINES

FIGURE 4.38 - LATERAL DEFLECTIONS OF TOP CHORDS
JULY TEST



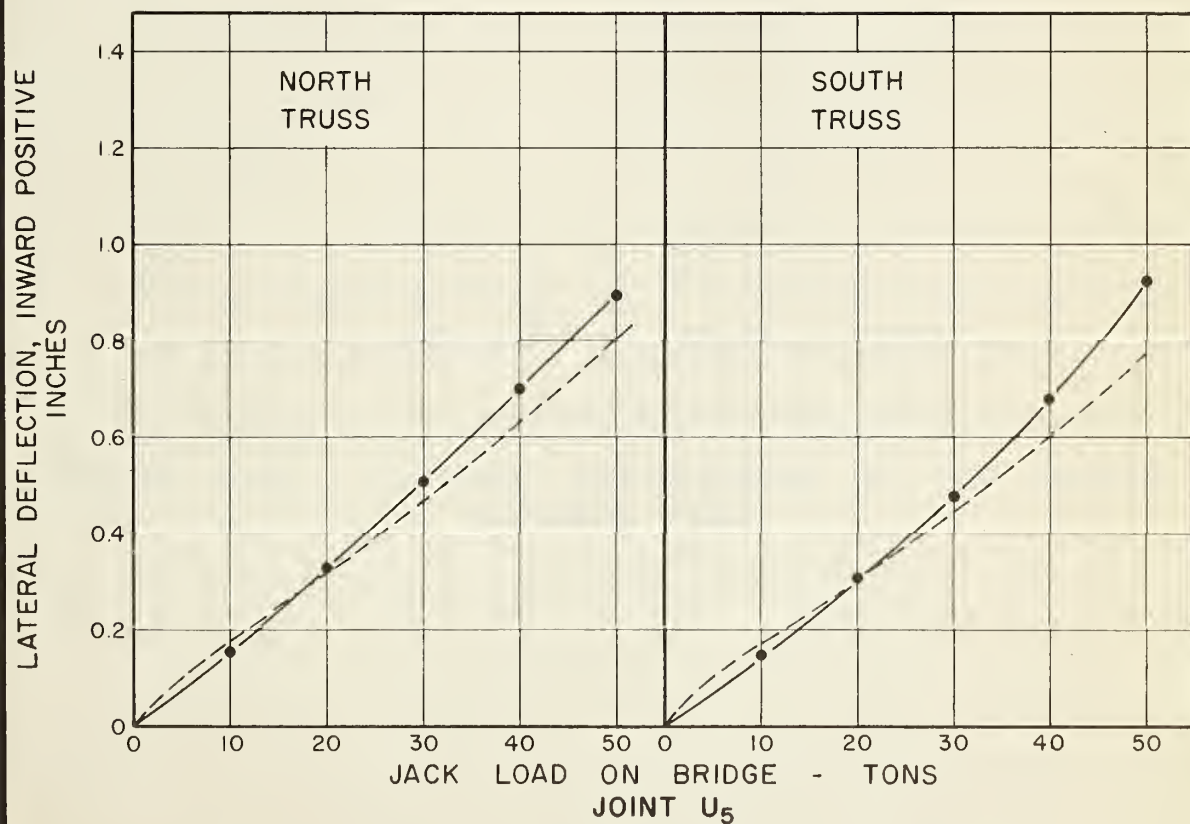
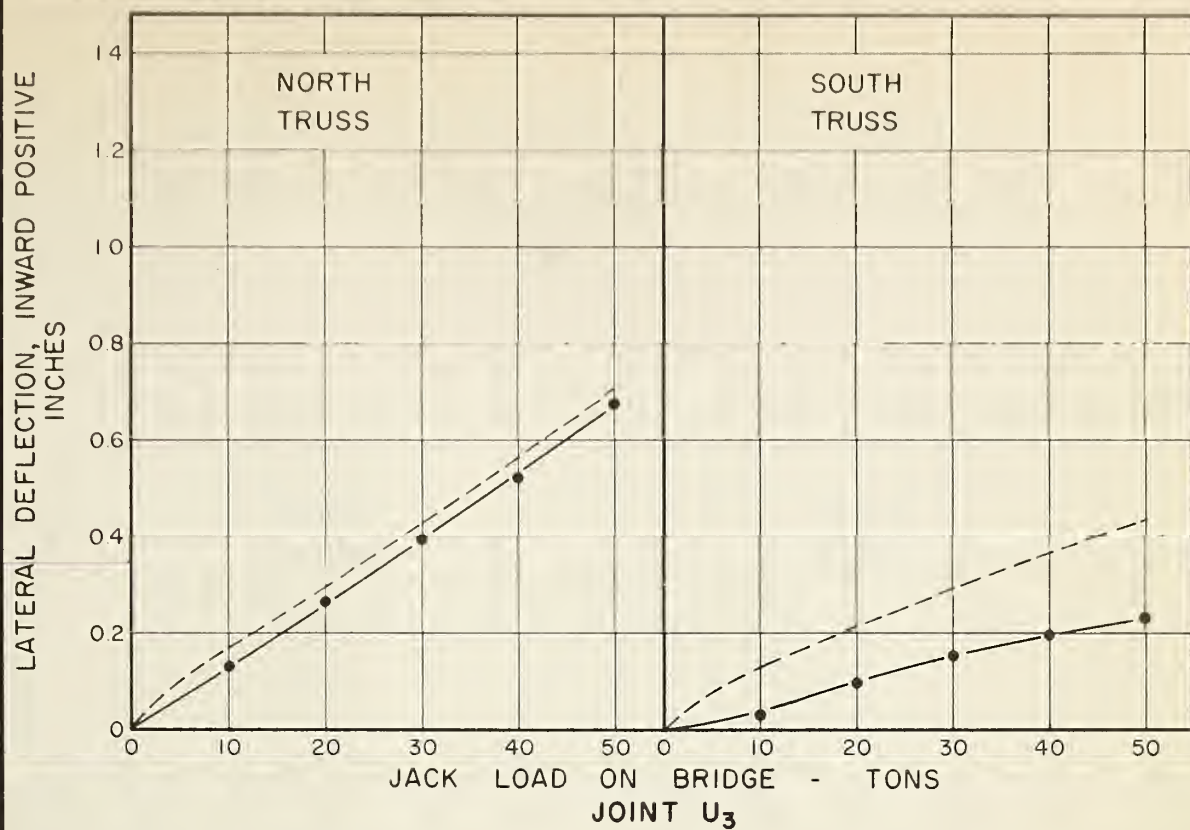
THEORETICAL DEFLECTIONS ILLUSTRATED BY BROKEN LINES

FIGURE 4.39 - LATERAL DEFLECTIONS OF TOP CHORDS
JULY TEST



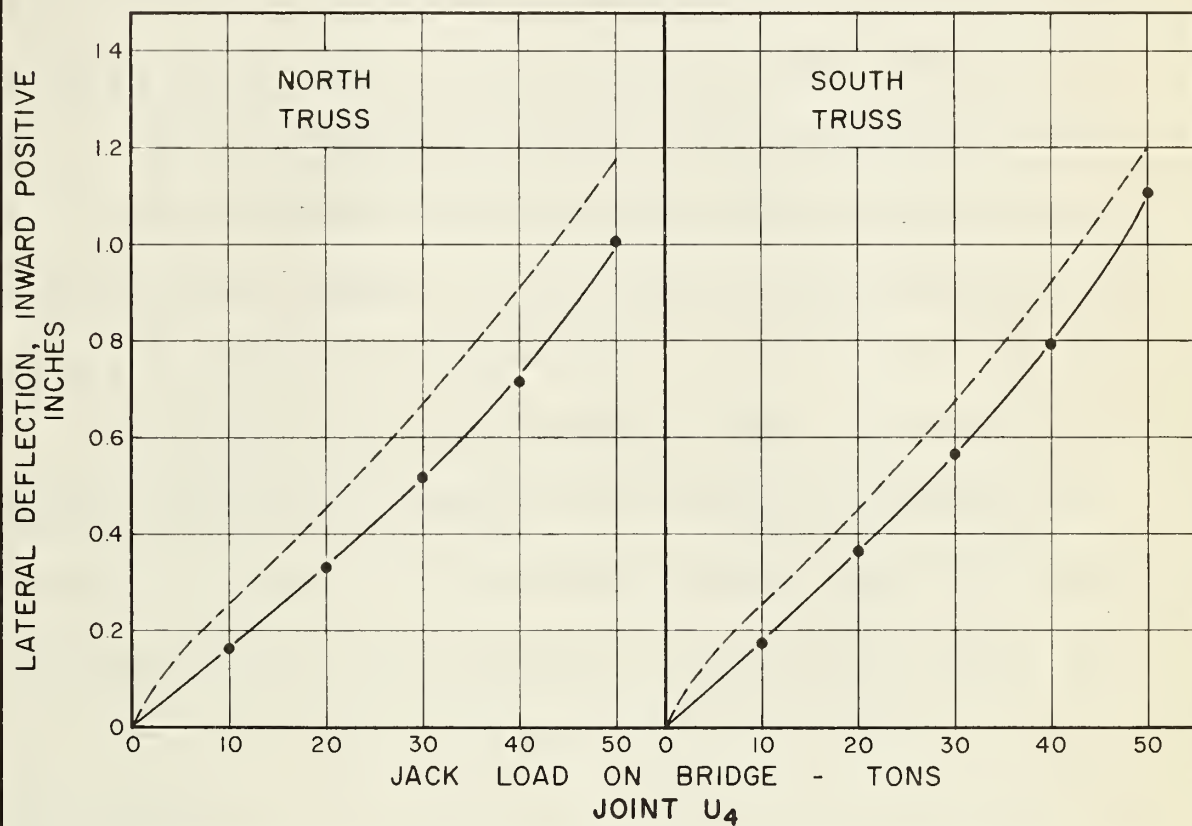
THEORETICAL DEFLECTIONS ILLUSTRATED BY BROKEN LINES

FIGURE 4.40 - LATERAL DEFLECTIONS OF TOP CHORDS
OCTOBER TEST



THEORETICAL DEFLECTIONS ILLUSTRATED BY BROKEN LINES

FIGURE 4.41 - LATERAL DEFLECTIONS OF TOP CHORDS
OCTOBER TEST



THEORETICAL DEFLECTIONS ILLUSTRATED BY BROKEN LINES

FIGURE 4.42 - LATERAL DEFLECTIONS OF TOP CHORDS
OCTOBER TEST

Thus an anomaly exists within the slope-deflection equations. On the one hand, the axial force terms in the equations must include the effects of the dead load, since the stiffness coefficients of a member depend upon the total axial force in that member. On the other hand, the initial top chord deflections include the dead load deflections, and computation of live load deflections naturally dictates that the axial force terms in the equilibrium equations must not include the effects of the dead load. Due to lack of sufficient memory storage capacity in the University computer, it was decided to defer modification of the analysis to eliminate this contradiction.

One other weakness in the analysis might be mentioned.

Whereas initial lateral deflections have been taken into account, no attempt has been made to include initial rotational displacements about the pertinent axes. Admittedly this is a more difficult problem, but it is felt that these rotational displacements might be at least approximated.

It is difficult to assess the influence of the shortcomings described above. It is hoped that their correction would narrow the already small gap between the theoretical and experimental results.

Measured deflections. The measured lateral deflections are also shown in Figures 4.37 through 4.42. In spite of the drawbacks noted in the foregoing section, the measured deflections are seen to be in generally good agreement with the computed deflections. It would

appear that only slight modifications to the analysis would be required to provide almost perfect correspondence.

There is no way of knowing whether or not the computed rotational displacements are accurate. It is presumed that the same precision could be expected in their calculation as in the calculation of the lateral deflections.

Because of the lack of even better theoretical and experimental conformity, no attempt was made to compute secondary stresses, although these would have proved of particular interest in the case of the vertical members. It was felt that until the corrections have been made to both theory and experimental procedure, such computations would be unwarranted.

CHAPTER V

SUMMARY AND RECOMMENDATIONS

1. Collapse of a pony-truss bridge may have numerous causes; buckling of a top chord is only one criterion.
2. Proper analysis of a pony-truss bridge must include the evaluation of secondary stresses, since failure may be initiated by local overstressing.
3. The collapse load in the July test was found to be 45 tons. Collapse of the bridge was immediately due to buckling of the top chord of the south truss. Formation of a plastic hinge in the center floor beam caused the chord to buckle prematurely.
4. The collapse load in the October test was found to be 65 tons. Collapse of the bridge was immediately due to buckling of the compression diagonals adjacent to the center panel point. Flexural yielding of the top chord of the south truss caused these diagonals to become greatly over-stressed. The chord itself did not buckle.
5. Only a negligible proportion of the load applied directly over a given floor beam was transferred through the wooden bridge deck

to adjacent beams.

6. The fact that the south chord of the July test buckled elastically tends to confirm the validity of Southwell's theory as applied to the problem by Lukawitski⁽⁸⁾.
7. The close correspondence between the 1959 and July, 1960 failure loads of 42 and 45 tons, respectively, demonstrates that the buckling load of the top chord appears to be virtually unaffected by its initial deflections, since the south chord was in a severely damaged condition prior to the 1959 test⁽⁸⁾.
This conclusion is sustained by the fact that no use is made of initial deflections in the computation of the buckling load by the analysis of Chapter II.
8. The slope-deflection analysis by Lee and Clough offers promise as a method of computing secondary stresses. The theory should be modified to allow for the correct use of live and dead loads, respectively, and if possible, for the consideration of initial rotational displacements.
9. There is ample scope for future load tests:
 - (a) A large variety of load distributions is possible.
 - (b) Such load tests should be designed to permit evaluation of the secondary stresses in the bridge.
 - (c) Some thought should be given to a more accurate

determination of floor beam bending moments.

- (d) Initial lateral deflections should be measured relative to the true ends of the top chord.
- (e) The behavior of single and especially double-angle members acting in both tension and compression should be thoroughly investigated in the laboratory before actual testing of a bridge.
- (f) Testing should be carried out under conditions more conducive to precision. That is, the test bridge should most certainly be erected indoors, in order to avoid the adverse effects of weather, and to permit preparation and testing on a reasonable schedule.

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BIBLIOGRAPHY

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APPENDICES

APPENDIX A

DIGITAL COMPUTER PROGRAM

The slope-deflection analysis by Lee and Clough has been programmed for the IBM 1620 electronic digital computer. The program follows the analysis as given in Chapter II, with the refinement that the total forces in the members are used to compute the stiffness factors, while only the live load forces are used in the slope-deflection equations.

Because of memory storage limitations, the program has been broken down into three shorter programs, the output of the first and second being used as input for the second and third, respectively. All three programs have been written in the IBM 1620 Fortran II compiler language. The source programs may be found at the end of this Appendix.

Definitions of Fortran II symbols used. Where possible, the Fortran II symbols have been defined in terms of the notation used in Chapter II. Wherever a symbol has two definitions, the second is possible because the first has been used for only a part of the program. This procedure results in a small saving in storage requirements.

Data required. The data to be prepared on cards comprises, in order:

1. the total number of members (always exclusive of the bottom chord) in the bridge to be analyzed;
2. the numerical designation of each member and the dead load force in that member;
3. identifying information;
4. the modulus of elasticity, the shear modulus, the number of joints (including the end joints), and the flexural and torsional fixity coefficients of the end joints;
5. the numerical designation of each member, its live load force, its length, its moment of inertia, its torsional rigidity and the angle which it makes with the horizontal;
6. the initial deflections, in ascending order of joint designation, of the top chord joints, exclusive of the end joints (assumed to be zero); and
7. the modified stiffness for antisymmetrical bending of each floor beam, and the fixed-end moment of each floor beam, alternately in ascending order of joint designation and exclusive of the end joints.

The data of parts 2 and 5 above are arranged such that there is one card for each member, in ascending order of joint designation.

First program. After reading sufficient information, the first program punches a card containing the same identifying information supplied as input, and a second card containing the modulus of elasticity, the shear modulus, the number of members, the number of joints, and the flexural and torsional fixity coefficients of the end joints. It then adds the live and dead load forces in each member, computes the stiffness parameters (ψ and ζ) and the carry-over factor (C) and punches two cards per member containing the following

information:

1. the numerical designation of the member, its live load force, its length, its moment of inertia, and its carry-over factor; and
2. the stiffness parameters, ψ and ζ , of the member, its torsional rigidity, and the angle it makes with the horizontal.

Finally, the program reads the initial deflections, floor beam stiffnesses and fixed-end moments, and punches this information in a format identical to that of the input data.

In summary, this program merely computes and inserts the stiffness parameters and carry-over factor into the list of data for each member, without otherwise altering the original list.

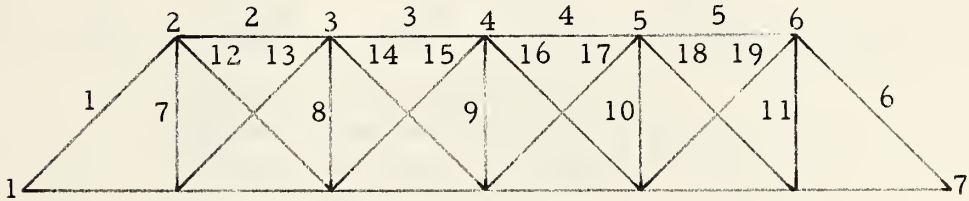
Second program. The output of the first program forms the input deck for the second program. After reading this deck, the program first punches a card containing the identification of the data. Then it computes and punches a number representing the required number of rows in the matrix of the slope-deflection equations, or in other words, the number of equations. The matrix is then punched by rows, four elements per card, until all the rows have been punched.

Third program. The third program uses as input the cards punched by the second program. It punches the identifying information, the determinant of the coefficients, and, if desired, the joint displacements.

I. JOINT AND MEMBER DESIGNATION

The joints and members must be assigned numerical designations according to the system used in Chapter II:

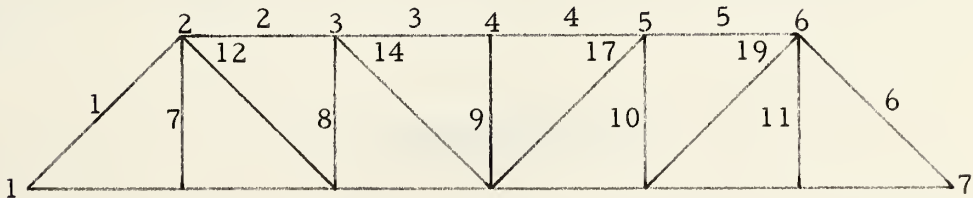
1. The upper chord joints are numbered consecutively from "1" at the left reaction.
2. The members are numbered consecutively from left to right, starting with the top chord members, continuing through the verticals, and finishing with the diagonals. The diagonals must be numbered consecutively at the top chord joints as though every panel contained two diagonals. If a panel contains only one diagonal, then the number of the missing diagonal is omitted. A few examples will help to clarify these instructions:



Number of joints (NJ) = 7

Number of members (NM) = 19

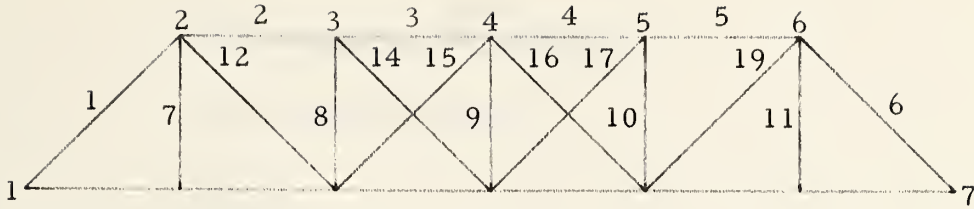
Maximum number of members = 19



Number of joints (NJ) = 7

Number of members (NM) = 15

Maximum number of members = 19

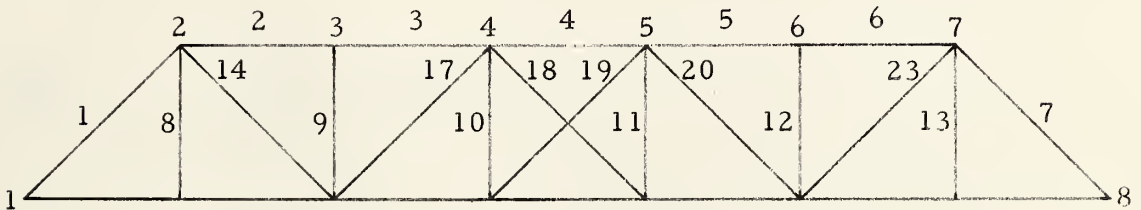


Number of joints (NJ) = 7

Number of members (NM) = 17

Maximum number of members = 19

Above example identical to test bridge



Number of joints (NJ) = 8

Number of members (NM) = 19

Maximum number of members = 23

Limitations. The size of truss is limited to one containing eleven joints and hence, thirty-five members, exclusive of the bottom chord.

II. PROGRAM NO. 1

The first program carries the University of Alberta Computing Center designation 920101A.

List of symbols. All quantities are in kips and inches.

A	a
B	b
C	C for compression member
CH	C for tension member
COSH	$\cosh \phi$
DELTA(M)	δ' for joint M
E	E
FEM(M)	M_f for joint M
FI	I
FK2B	K''_b for joint M
FL	L
G	G
I	index
J	member designation
NJ	number of joints
NM	number of members
P	live load plus dead load force in member
PDEAD(J)	dead load in member J
PHI	ϕ
PLIVE	P
PSI	ψ for compression member
PSIH	ψ for tension member
R	R

SINH	$\sinh \phi$
THETA	θ
ZETA	ζ for compression member
ZETAH	ζ for tension member

Data preparation. Data formats are indicated in IBM 1620

Fortran II notation.

Card(s) (in order)	Symbol	Format	Card Columns
1	NM	I3	1-3
2→NM+1	J	I3	1-3
	PDEAD(J)	F10.2	5-14
NM+2	identification	72H	1-72
NM+3	E	F10.0	1-10
	G	F10.0	13-22
	NJ	I3	25-27
	A	F5.1	30-34
	B	F5.1	37-41
NM+4			
→2NM+3	J	I3	1-3
	PLIVE	F10.2	5-14
	FL	F10.2	16-25
	FI	F10.2	27-36
	R	F10.2	38-47
	THETA	F10.4	49-58
Additional as required	DELTA(M)	F7.3, 8F8.3	1-7, 8-15, etc.
Additional as required	FK2B(M) } FEM(M) }	(4(F11.1, F7.1))	1-11, 19-29, etc. 12-18, 30-36, etc.

Sample data as punched on cards are shown in Table A.1.

Operating instructions.

1. Clear core storage:
 - a) RESET
 - b) INSERT
 - c) Type 16 00010 00000
 - d) RELEASE
 - e) START
 - f) INSTANT STOP
 - g) RESET
2. Load object deck - push LOAD key.

3. Set switches:

Switch 1

OFF to process one set of data. The entire program (requiring dead load data) may be re-executed by depressing START.

ON to process several sets of data using the same dead load forces. The first NM+1 cards must be entered only once, at the start of the run. As many sets of data cards (NM+2 through 2NM+3 + initial deflections, beam stiffnesses and fixed-end moments) as desired may then be entered in sequence. Switch should be turned OFF after the last set of data has been read.

4. Load data.
5. Push START.

Output. Output is by cards, and has been described previously.

The output from the sample data of Table A.1 is given in Table A.2, as printed from cards.

III. PROGRAM NO. 2

The second program carries the University of Alberta

Computing Center designation 920101B.

List of symbols. All quantities are in kips and inches.

A(1, J)	P for member J
A(2, J)	L of member J
A(3, J)	I of member J
A(4, J)	C of member J
A(5, J)	ψ for member J
A(6, J)	ζ for member J
A(7, J)	R of member J
A(8, J)	θ of member J
All, A21, etc.	coefficients of slope-deflection equations
AA	"a" for left end of truss
AB	"a" for right end of truss
AF	"a" input data for both ends of truss
B	"b" input data for both ends of truss
BA	"b" for right end of truss
BB	"b" for left end of truss
CDL	$\cos \theta_d$ for diagonal to left of joint
CDR	$\cos \theta_d$ for diagonal to right of joint
CN	$\cos \theta_n$
CNM1	$\cos \theta_{n-1}$
DELTA(I)	δ' of joint I
E	E

FEM(I)	M_f for beam I
FJ	$1 - aC_n$
FJ1	$1 - aC_n^2$
FK	K $1 - aC_{n-1}$
FK1	K' $1 - aC_{n-1}^2$
FK2B(I)	K''_b for beam I
FKDL	K_d for diagonal to left of joint
FK1DL	K'_d for diagonal to left of joint
FKDR	K_d for diagonal to right of joint
FK1DR	K'_d for diagonal to right of joint
FKN	K_n
FK1N	K'_n
FKNM	K_{n-1}
FK1NM	K'_{n-1}
FKV	K_v
FK1V	K'_v
G	G
H(I, J)	element of output matrix
I	index
IXZ	index
J	member designation during input index
JDL	subscript "d" for diagonal to left of joint

JDR	subscript "d" for diagonal to right of joint
JJ	index
JQ	index
JV	subscript "v"
M	index
MNM	maximum number of members + 1
N	subscript "n"
NC	number of columns in matrix
NJ	number of joints
NM	number of members
NM1	subscript "n - 1"
NMEMB(I)	member designation storage array
NN	number of joints - 1
NP1	subscript "n + 1"
NR	number of equations to be established (i.e. number of rows in matrix)
PDL	P_d for diagonal to left of joint
PN	$ P_n $
PNM1	$ P_{n-1} $
RDL	R_d for diagonal to left of joint
RR	r $l-b$
RS	$l-b$
SDL	$\sin \theta_d$ for diagonal to left of joint

SDR	$\sin\theta_d$ for diagonal to right of joint
SN	$\sin\theta_n$
SNM1	$\sin\theta_{n-1}$
SX	S_x
S1XX	S'_{xx}
S1XZ	S'_{xz}
S1YY	S'_{yy}
SZ	S_z
S1ZZ	S'_{zz}
S2XX	S''_{xx}
S2XY	S''_{xy}
S2XZ	S''_{xz}
S2YY	S''_{yy}
S2YZ	S''_{yz}
S2ZZ	S''_{zz}
TS	temporary storage

Data preparation. The cards punched by the previous program (see Table A.2) are used directly as input for this program.

Operating instructions.

1. Clear core storage (as for Program No. 1).
2. Load object deck - push LOAD key.
3. Set switches:

Switch 1

OFF to process one set of data. Program may be re-executed by pushing START.

ON to process more than one set of data. Switch 1 may be turned off after last set of data has been read.

4. Load data.
5. Push START.

Output. Output is by punched cards, and has been described previously. The output from the data of Table A.2 is given in Table A.3, and is used directly as input for Program No. 3.

IV. PROGRAM NO. 3

The third program carries the University of Alberta Computing Center designation 920101C.

List of symbols. The equation solving routine incorporated into this program was supplied by the University of Alberta Computing Center. Since it was known to be in satisfactory working order, it was not analyzed by the author. As a result, certain symbols used only in this routine have not been defined.

A(I, J)	matrix storage array - working area
C	multiplying factor
D	value of determinant
FLO	used in round-off routine
FMOD	matrix element modifier

H(I, J)	matrix storage array - permanent storage
I	index output control index
II	output control index
III	output control index
IA	joint number (in output routine)
INTEG	used in roundoff routine
J	index
JKL	path control index
JQ	number of columns in matrix
K	index
M	index
N	number of equations, or rows in matrix
NJ	number of joints
PAR	used in roundoff routine

Data preparation. The cards punched by Program No. 2 are used directly as input for this program.

Operating instructions.

1. Clear core storage (as for Program No. 1).
2. Load object deck - push LOAD key.
3. Set switches:

Switch 1

OFF to process only one set of data; also off to insert new

matrix modifier (see Switch 3). Program may be re-executed by pushing START.

ON to process more than one set of data.

Switch 2

OFF to compute only determinant.

ON to compute determinant and displacements.

Switch 3

This switch is normally OFF. It may happen that the value of the determinant exceeds the range of floating point numbers available. A method has been provided whereby if this occurs, each element of the matrix may be multiplied by a common factor, in order to bring the value of the determinant into the admissible range. The data is reloaded, Switch 3 is turned ON, and the program is restarted. This may be done after turning off Switch 1 and waiting for the computer to stop in manual, then then pushing START, or by branching to the first instruction of the program. The typewriter will be activated, and the operator may type in any suitable modifier, in floating point format. The RELEASE and START keys are then depressed, and the program will continue, using the modified matrix for the determinant calculation only. This procedure may be repeated as many times as necessary. The final answer must be corrected to take the modifier into account.

Switch 1 may now be turned ON or OFF. If it is ON, the modifier will remain unchanged. If it is OFF, the entire program will return to its initial condition, and a new modifier may be inserted. The modifier routine may be bypassed at any time, without altering the modifier, by turning Switch 3 OFF, as long as Switch 1 is ON.

5. Load data.

6. Push START.

Output. Output is by cards. The identification card is punched first, then the determinant. If the equations have been solved, suitable

headings are punched, then the joint number, δ , α and β for each joint. The output from the data of Table A.3 is given in Table A.4.

V. SOURCE PROGRAM NO. 1

```

        DIMENSION PDEAD(35),DELTA(11),FK2B(10),FEM(10)
27  PRINT 201
    PRINT 202
    PRINT 204
201  FORMAT (35H SWITCH 1 ON TO CONTINUE DATA INPUT,)
202  FORMAT (30H OFF AFTER LAST DATA CARD READ.)
204  FORMAT(11HPUSH START.)
    PAUSE
    READ 798, NM
798  FORMAT (I3)
    DO 800 I=1,NM
800  READ 700,J,PDEAD(J)
700  FORMAT (I3,1X,F10.2)
    5 READ 711
711  FORMAT(72H
1      )
    PUNCH 711
    READ 109,E,G,NJ,A,B
109  FORMAT (F10.0,2X,F10.0,2X,I3,2X,F5.1,2X,F5.1)
    PUNCH 110,E,G,NM,NJ,A,B
110  FORMAT (F10.0,2X,F10.0,2X,I4,2X,I3,2X,F5.1,2X,F5.1)
    I=1
    24 READ 111,J,PLIVE,FL,FI,R,THETA
111  FORMAT (I3,1X,F10.2,1X,F10.2,1X,F10.2,1X,F10.2,1X,F10.4)
    P=PLIVE+PDEAD(J)
    2 IF (P) 6,15,11
    6 P=-P
    PHI = FL*SQRTF(P/E/FI)
    P=-P
    IF (PHI-.2) 87,87,88
87  ZETA = (120.-6.*PHI**2)/(120.-20.*PHI**2+PHI**4)
    PSI = (120.-12.*PHI**2)/(120.-20.*PHI**2+PHI**4)
    GO TO 89
88  ZETA = 6.*(PHI/SINF(PHI)-1.)/PHI**2
    PSI = 3.*(1.-PHI*COSF(PHI)/SINF(PHI))/PHI**2
89  C = ZETA/2./PSI
    GO TO 20
11  PHI = FL*SQRTF(P/E/FI)
    IF (PHI-.2) 97,97,99
97  ZETAH = (120.+6.*PHI**2)/(120.+20.*PHI**2+PHI**4)
    PSIH = (120.+12.*PHI**2)/(120.+20.*PHI**2+PHI**4)
    GO TO 98
99  SINH = (EXPF(PHI)-EXPF(-PHI))/2.
    COSH = (EXPF(PHI)+EXPF(-PHI))/2.
    ZETAH = 6.*(1.-PHI/SINH)/PHI**2
    PSIH = 3.*(PHI*COSH/SINH-1.)/PHI**2
98  CH = ZETAH/2./PSIH
20  IF (P) 21,21,22
15  ZETA = 1.0

```



```
PSI = 1.0
C = .5
21 PUNCH 115,J,PLIVE,FL,FI,C
115 FORMAT (I3,1X,F10.2,1X,F10.2,1X,F10.2,1X,E14.8)
PUNCH 116,PSI,ZETA,R,THETA
116 FORMAT (E14.8,1X,E14.8,1X,F10.2,1X,F10.4)
GO TO 23
22 PUNCH 115,J,PLIVE,FL,FI,CH
PUNCH 116,PSIH,ZETAH,R,THETA
23 I=I+1
IF (I-NM) 24,24,32
32 NN=NJ-1
READ 702,(DELTA(M),M=2,NN)
702 FORMAT(F7.3,8F8.3)
READ 703,(FK2B(M),FEM(M),M=2,NN)
703 FORMAT(4(F11.1,F7.1))
PUNCH 702,(DELTA(M),M=2,NN)
PUNCH 703,(FK2B(M),FEM(M),M=2,NN)
IF (SENSE SWITCH 1) 5,31
31 PRINT 119
119 FORMAT (32H MAY RESTART PROGRAM. PUSH START./)
PAUSE
GO TO 27
END
```


VI. SOURCE PROGRAM NO. 2

```

C      PROGRAM NO. 920101B-3
C      SLOPE-DEFLECTION ANALYSIS OF A PONY-TRUSS.
      DIMENSION A(8,36),H(27,28),NMEMB(35),DELTA(11),FK2B(10),FEM(10)
C      INPUT OF DATA
100 READ 711
711 FORMAT(72H
      1
      )
      PUNCH 711
      READ 700,F,G,NM,NJ,AF,B
700 FORMAT(F10.0,2X,F10.0,2X,I4,2X,I3,2X,F5.1,2X,F5.1)
      AA=AF
      BB=B
      AB=0.0
      BA=0.0
      DO 101 M=1,NM
      READ 701,J,(A(I,J),I=1,8)
701 FORMAT(I3,3F11.2,E15.8/E14.8,E15.8,F11.2,F11.4)
101 NMEMB(M)=J
      NN=NJ-1
      DELTA (1) = 0.0
      DELTA(NJ)=0.0
102 READ 702,(DELTA(M),M=2,NN)
702 FORMAT(F7.3,8F8.3)
103 READ 703,(FK2B(M),FEM(M),M=2,NN)
703 FORMAT(4(F11.1,F7.1))
C      END OF DATA INPUT.
C      DETECT AND ELIMINATE MISSING MEMBERS.
      MNM=4*NJ-8
      DO 104 J=1,MNM
      DO 105 M=1,NM
      IF (NMEMB(M)-J) 105,104,105
105 CONTINUE
      A(1,J)=0.0
      A(2,J)=1.0
      A(3,J)=0.0
      A(4,J)=0.0
      A(5,J)=0.0
      A(6,J)=1.0
      A(7,J)=0.0
      A(8,J)=0.0
104 CONTINUE
C      COMPUTE AND PUNCH NUMBER OF ROWS IN MATRIX.
      NR=3*(NJ-2)
      NC=NR+1
      PUNCH 705,NR
705 FORMAT(I5)
C      THE SLOPE-DEFLECTION COEFFICIENTS ARE COMPUTED AND PUNCHED.
      DO 121 I=1,27
      DO 121 J=1,28

```



```

121 H(I,J)=0.0
    IXZ=1
    NM1=1
    N=2
    JJ=1
    NP1=3
    JV=NJ
    JDR=2*NJ-2
    M=0
    JQ=NM1
    GO TO 106
108 FKNM=FK
    FK1NM=FK1
109 JQ=N
    GO TO 106
110 FKN=FK
    FK1N=FK1
    JQ=JV
    GO TO 106
111 FKV=FK
    FK1V=FK1
    JQ=JDR
    GO TO 106
112 FKDR=FK
    FK1DR=FK1
    JDL=JDR-1
115 IF (N-2) 113,116,113
113 JQ=JDL
106 FK=E*A(3,JQ)*12.*A(5,JQ)/(A(2,JQ)*((4.*A(5,JQ)**2)-A(6,JQ)**2))
    FK1=(1.+A(4,JQ))*FK
    M=M+1
    GO TO (108,110,111,112,114),M
116 FKDL=0.0
    FK1DL=0.0
    RDL=0.0
    PDL=0.0
    GO TO 117
114 FKDL=FK
    FK1DL=FK1
    RDL=A(7,JDL)
    PDL=A(1,JDL)
117 RR=FK2B(N)/FKV
    SDR=SINF(A(8,JDR))
    CDR=COSF(A(8,JDR))
    SDL=SINF(A(8,JDL))
    CDL=COSF(A(8,JDL))
    SNM1=SINF(A(8,NM1))
    CNM1=COSF(A(8,NM1))
164 SN=SINF(A(8,N))
    CN=COSF(A(8,N))
    S1XX=(1.-(A(4,JV)**2/(1.+RR)))*FKV
    S1XZ=(1.-(A(4,JV)/(1.+RR)))*(FK1V/A(2,JV))

```



```

S1YY=A(7,JV)
TS=A(1,JV)/A(2,JV)
S1ZZ=(2.-(1.+A(4,JV))/(1.+RR))*(FK1V/(A(2,JV)**2))+TS
S2XX=FKDR*SDR**2+A(7,JDR)*CDR**2+FKDL*SDL**2+RDL*CDL**2
S2XY=((FKDR-A(7,JDR))*SDR*CDR)-((FKDL-RDL)*SDL*CDL)
S2XZ=((FK1DR/A(2,JDR))*SDR)+((FK1DL/A(2,JDL))*SDL)
S2YY=FKDR*CDR**2+A(7,JDR)*SDR**2+FKDL*CDL**2+RDL*SDL**2
S2YZ=((FK1DR/A(2,JDR))*CDR)-((FK1DL/A(2,JDL))*CDL)
TS=(2.*FK1DL/(A(2,JDL)**2))+(PDL/A(2,JDL))
S2ZZ=(2.*FK1DR/(A(2,JDR)**2))+(A(1,JDR)/A(2,JDR))+TS
SX=-(FEM(N)*A(4,JV)/(1.+RR))
SZ=-(FEM(N)/A(2,JV)*(1.+A(4,JV))/(1.+RR))
A11=-((FK1NM/A(2,NM1))*SNM1)
A21=FK1NM/A(2,NM1)*CNM1
IF (A(1,NM1)) 500,501,501
500 PNM1=-(A(1,NM1))
GO TO 502
501 PNM1=A(1,NM1)
502 IF (A(1,N)) 503,504,504
503 PN=-(A(1,N))
GO TO 505
504 PN=A(1,N)
505 IF (N-2) 119,120,119
119 A12=A(4,NM1)*FKNM*SNM1**2-A(7,NM1)*CNM1**2
A13=-((A(4,NM1)*FKNM+A(7,NM1))*SNM1*CNM1)
A23=A(4,NM1)*FKNM*CNM1**2-A(7,NM1)*SNM1**2
A31=PNM1/A(2,NM1)-2.*FK1NM/(A(2,NM1)**2)
A32=-A11
A33=-A21
120 A17=FK1N/A(2,N)*SN
FK=1.-AA*A(4,NM1)
FJ=1.-AB*A(4,N)
A14=FK*(-A11)-FJ*A17+S1XZ+S2XZ
FK1=1.-AA*A(4,NM1)**2
FJ1=1.-AB*A(4,N)**2
RR=1.-BB
RS=1.-BA
TS=RS*A(7,N)*CN**2+S1XX+S2XX
A15=FK1*FKNM*SNM1**2+RR*A(7,NM1)*CNM1**2+FJ1*FKN*SN**2+TS
TS=S2XY
A16=(RR*A(7,NM1)-FK1*FKNM)*SNM1*CNM1+(RS*A(7,N)-FJ1*FKN)*SN*CN+TS
A27=-((FK1N/A(2,N))*CN)
A24=S2YZ-A21*FK-A27*FJ
TS=A(7,N)*RS*SN**2+S1YY+S2YY
A26=FKNM*FK1*CNM1**2+A(7,NM1)*RR*SNM1**2+FKN*FJ1*CN**2+TS
TS=S1ZZ+S2ZZ-PN/A(2,N)+(2.-AB*(1.+A(4,N)))*FK1N/A(2,N)**2
A34=(2.-AA*(1.+A(4,NM1)))*FK1NM/A(2,NM1)**2-PNM1/A(2,NM1)+TS
IF (NN-N) 300,301,300
300 A18=A(4,N)*FKN*SN**2-A(7,N)*CN**2
A19=-((A(4,N)*FKN+A(7,N))*SN*CN)
A29=A(4,N)*FKN*CN**2-A(7,N)*SN**2
A37=PN/A(2,N)-((2.*FK1N)/(A(2,N)**2))

```



```

A38=-A17
A39=-A27
301 TS=(DELTA(N)-DELTA(NM1))*(1.-AA*A(4,NM1))*FK1NM*SNM1/A(2,NM1)
TS=(DELTA(NP1)-DELTA(N))*(1.-AB*A(4,N))*FK1N*SN/A(2,N)+TS
H(IXZ,NC) = DELTA(N)*(S1XZ+S2XZ)-SX+TS
TS=(DELTA(NM1)-DELTA(N))*(1.-AA*A(4,NM1))*FK1NM*CNM1/A(2,NM1)
TS=TS-(DELTA(NP1)-DELTA(N))*(1.-AB*A(4,N))*FK1N*CN/A(2,N)
H(IXZ+1,NC) = TS+DELTA(N)*S2YZ
TS=(2.-AA*(1.+A(4,NM1)))*(DELTA(N)-DELTA(NM1))*FK1NM/A(2,NM1)**2
TS=TS-(2.-AB*(1.+A(4,N)))*(DELTA(NP1)-DELTA(N))*FK1N/A(2,N)**2
H(IXZ+2,NC) = TS+DELTA(N)*(S1ZZ+S2ZZ)-SZ
124 J=JJ
I=IXZ
IF (N-2) 122,123,122
122 H(I,J)=A11
J=J+1
H(I,J)=A12
J=J+1
H(I,J)=A13
J=J+1
123 H(I,J)=A14
J=J+1
H(I,J)=A15
J=J+1
H(I,J)=A16
IF (NN-N) 125,126,125
125 J=J+1
H(I,J)=A17
J=J+1
H(I,J)=A18
J=J+1
H(I,J)=A19
126 I=IXZ+1
J=JJ
IF (N-2) 127,128,127
127 H(I,J)=A21
J=J+1
H(I,J)=A13
J=J+1
H(I,J)=A23
J=J+1
128 H(I,J)=A24
J=J+1
H(I,J)=A16
J=J+1
H(I,J)=A26
IF (NN-N) 129,130,129
129 J=J+1
H(I,J)=A27
J=J+1
H(I,J)=A19
J=J+1

```



```

      H(I,J)=A29
130  I=IXZ+2
      J=JJ
      IF (N-2) 131,132,131
131  H(I,J)=A31
      J=J+1
      H(I,J)=A32
      J=J+1
      H(I,J)=A33
      J=J+1
132  H(I,J)=A34
      J=J+1
      H(I,J)=A14
      J=J+1
      H(I,J)=A24
      IF (NN-N) 133,153,133
133  J=J+1
      H(I,J)=A37
      J=J+1
      H(I,J)=A38
      J=J+1
      H(I,J)=A39
134  N=N+1
      IF (N-NN) 135,303,153
303  AB=AF
      BA=B
135  JJ=3*N-8
      IXZ=IXZ+3
      M=1
      NM1=NM1+1
      NP1=NP1+1
      JV=JV+1
      JDR=JDR+2
      FKNM=FKN
      FK1NM=FK1N
      AA=0.0
      BB=0.0
      GO TO 109
153  PUNCH 704, ((H(I,J),J=1,NC),I=1,NR)
704  FORMAT(E15.8,3E19.8)
      IF (SENSE SWITCH 1) 100,200
C    CONSOLE PROGRAM SWITCH 1 ON TO INPUT MORE DATA
200  PRINT 714
714  FORMAT(22HTO RE-RUN, PUSH START.)
      PAUSE
      GO TO 100
      END

```


VII. SOURCE PROGRAM NO. 3

```

C   PROGRAM NUMBER 920101C-1. THIS PROGRAM IS TO BE USED WITH
C   PROGRAM NUMBER 920101B-3. THE MATRICES PUNCHED BY THAT PROGRAM
C   ARE USED DIRECTLY AS INPUT FOR THIS PROGRAM. THIS PROGRAM
C   COMPUTES THE DETERMINANT OF THE COEFFICIENTS OF A SET OF
C   SIMULTANEOUS EQUATIONS AND, IF DESIRED, ALSO SOLVES THE EQUATIONS.
    DIMENSION A(27,28),H(27,28),L(27)
198 JKL=0
199 READ 711
711 FORMAT(72H
      1
      )
      PUNCH 711
      PUNCH 712
712 FORMAT(5H
      )
      READ 705,N
705 FORMAT(15)
      JQ=N+1
      READ 704,((A(I,J),J=1,JQ),I=1,N)
704 FORMAT(E15.8,3E19.8)
      DO 200 I=1,N
      DO 200 J=1,JQ
200 H(I,J)=A(I,J)
      IF (SENSE SWITCH 3) 1,201
      1 IF (JKL) 201,5,201
      5 PRINT 713
713 FORMAT(55HTYPE SUITABLE MODIFIER(FORMAT F5.0), RELEASE AND START.)
      ACCEPT 706,FMOD
706 FORMAT(F5.0)
      2 DO 33 I=1,N
      DO 33 J=1,JQ
      33 A(I,J)=A(I,J)*FMOD
197 JKL=1
C   DETERMINANT CALCULATOR SUBROUTINE
201 K=N
      I=N
      J=N
      M=N
202 IF (A(M,M)) 209,203,209
203 I=I-1
204 IF (I) 207,205,207
205 D=0.0
      GO TO 224
207 IF (A(I,J)) 208,203,208
208 A(K,J)=A(I,J)+A(K,J)
210 J=J-1
211 IF (J) 208,219,208
209 I=I-1
212 IF (I) 213,228,213
213 J=M
      IF (A(I,J)) 214,209,214

```



```

214 C=A(I,J)/A(K,J)
215 A(I,J)=A(I,J)-C*A(K,J)
216 J=J-1
217 IF (J) 215,209,215
228 IF (M-1) 218,220,218
218 M=M-1
219 I=M
      J=M
      K=M
      GO TO 202
220 D=A(M,M)
221 M=M+1
222 D=A(M,M)*D
223 IF (M-N) 221,224,221
224 PUNCH 709,D
709 FORMAT(14HDETERMINANT = 1PE14.7//)
C     CONSOLE PROGRAM SWITCH 2 ON TO COMPUTE DETERMINANT AND DISPLACEMENT
C     CONSOLE PROGRAM SWITCH 2 OFF TO COMPUTE ONLY DETERMINANT
C     JORDAN'S METHOD OF SOLVING SIMULTANEOUS EQUATIONS
      IF (SENSE SWITCH 2) 142,206
142 DO 166 I=1,N
      DO 166 J=1,JQ
166 A(I,J)=H(I,J)
143 M=N+1
303 DO 315 K=1,N
      L(K)=K
      IF (A(K,K)) 311,304,311
C     SEARCH FOR NON-ZERO A(I,K)
304 I=K+1
305 IF (N-1) 306,307,307
306 PRINT 715
715 FORMAT(45HMATRIX SINGULAR - ENTER NEW DATA, PUSH START.)
      PAUSE
      GO TO 199
307 IF (A(I,K)) 309,308,309
308 I=I+1
      GO TO 305
C     CHANGE ROWS I AND K
309 DO 310 J=1,M
      TS=A(I,J)
      A(I,J)=A(K,J)
310 A(K,J)=TS
      L(K)=I
C     PERFORM ONE PIVOTAL OPERATION
311 PP=1.0/A(K,K)
      A(K,K)=1.0
      DO 312 J=1,M
312 A(K,J)=PP*A(K,J)
      DO 315 I=1,N
      IF (K-I) 313,315,313
313 B=A(I,K)
      A(I,K)=0.0E-49

```



```

      DO 314 J=1,M
314  A(I,J)=A(I,J)-B*A(K,J)
315  CONTINUE
C    REARRANGE MATRIX
      K=N
      DO 318 IA=1,N
      IF (L(K)-K) 316,318,316
C    CHANGE COLUMNS L(K) AND K
316  I=L(K)
      DO 317 J=1,N
      TS=A(J,I)
      A(J,I)=A(J,K)
317  A(J,K)=TS
318  K=K-1
C    PRINT JOINT NUMBER, DELTA, ALPHA, AND BETA.
      NJ=N/3+1
      I=1
      II=2
      III=3
      PUNCH 707
707  FORMAT(7X5HJOINT,6X5HDELTA,10X5HALPHA,15X4HBETA/7X6HNUMBER//)
      DO 321 IA=2,NJ
      PAR=A(I,M)
      DO 802 K=1,4
      INTEG = PAR
      FLO=INTEG
802  PAR=(PAR-FLO)*10.
      IF (A(I,M)) 806,805,805
805  IF (PAR-5.) 804,803,803
806  IF (PAR+5.) 807,807,804
803  A(I,M)=A(I,M)-PAR*10.**(-4)+10.**(-3)
      GO TO 804
807  A(I,M)=A(I,M)-PAR*10.**(-4)-10.**(-3)
804  PUNCH 708,IA,A(I,M),A(II,M),A(III,M)
708  FORMAT(9XI2,F12.3,1PE19.7,E20.7)
      I=I+3
      II=II+3
321  III=III+3
206  IF (SENSE SWITCH 1) 199,400
400  PRINT 714
714  FORMAT(22HTO RE-RUN, PUSH START.)
      PAUSE
      GO TO 198
      END

```


TABLE A.2 - OUTPUT OF PROGRAM NO. 1

SAMPLE DATA - SOUTH TRUSS, OCTOBER LOAD DISTRIBUTION, 30 TONS.

	30000.	12000.	17	7	1.0	0.0
1	-25.83	196.50		18.95	.59020112E+00	
.12867035E+01		.15188277E+01		20.60	.6190	
2	-42.08	160.00		18.95	.59099478E+00	
.12897332E+01		.15244512E+01		25.40	0.0000	
3	-47.67	160.00		18.95	.60435946E+00	
.13424503E+01		.16226451E+01		25.40	0.0000	
4	-47.67	160.00		18.95	.60435946E+00	
.13424503E+01		.16226451E+01		25.40	0.0000	
5	-42.08	160.00		18.95	.59099478E+00	
.12897332E+01		.15244512E+01		25.40	0.0000	
6	-25.83	196.50		18.95	.59020112E+00	
.12867035E+01		.15188277E+01		20.60	-.6190	
7	0.00	125.00		60.60	.49937136E+00	
.99832131E+00		.99706617E+00		35.90	1.5708	
8	-3.98	125.00		60.60	.50135565E+00	
.10036464E+01		.10063676E+01		35.90	1.5708	
9	4.46	125.00		60.60	.49877643E+00	
.99671192E+00		.99427284E+00		35.90	1.5708	
10	-3.98	125.00		60.60	.50135565E+00	
.10036464E+01		.10063676E+01		35.90	1.5708	
11	0.00	125.00		60.60	.49937136E+00	
.99832131E+00		.99706617E+00		35.90	1.5708	
12	25.83	196.50		4.23	.31893855E+00	
.62128349E+00		.39630253E+00		5.65	.6190	
14	6.83	196.50		.90	.29659397E+00	
.58351596E+00		.34613464E+00		3.13	.6190	
15	-3.83	196.50		.85	.85230095E+00	
.39583601E+01		.67474282E+01		2.83	.6190	
16	-3.83	196.50		.85	.85230095E+00	
.39583601E+01		.67474282E+01		2.83	.6190	
17	6.83	196.50		.90	.29659397E+00	
.58351596E+00		.34613464E+00		3.13	.6190	
19	25.83	196.50		4.23	.31893855E+00	
.62128349E+00		.39630253E+00		5.65	.6190	
-.060	-.730	-.110	-.210	-.660		
126229.0	0.0	126229.0	421.0	126229.0	594.0	126229.0 421.0
126229.0	0.0					

TABLE A.3 - OUTPUT OF PROGRAM NO. 2

SAMPLE DATA - SOUTH TRUSS, OCTOBER LOAD DISTRIBUTION, 30 TONS.

15

.62175387E+03	.57124167E+05	-.15400826E+04	.00000000E-99
-.25400000E+02	-.00000000E-99	.00000000E-99	.00000000E-99
.00000000E-99	.00000000E-99	.00000000E-99	.00000000E-99
.00000000E-99	.00000000E-99	.00000000E-99	-.37305231E+02
.11731337E+03	-.15400826E+04	.19522989E+05	-.12629408E+03
-.00000000E-99	.75061614E+04	.00000000E-99	.00000000E-99
.00000000E-99	.00000000E-99	.00000000E-99	.00000000E-99
.00000000E-99	.00000000E-99	.00000000E-99	.85155873E+02
.10257643E+02	.62175387E+03	.11731337E+03	-.13156760E+01
-.00000000E-99	.12629408E+03	.00000000E-99	.00000000E-99
.00000000E-99	.00000000E-99	.00000000E-99	.00000000E-99
.00000000E-99	.00000000E-99	.00000000E-99	.51330780E+00
-.00000000E-99	-.25400000E+02	-.00000000E-99	.59025372E+03
.53784326E+05	.36455565E+03	.00000000E-99	-.25400000E+02
-.00000000E-99	.00000000E-99	.00000000E-99	.00000000E-99
.00000000E-99	.00000000E-99	.00000000E-99	-.36438002E+03
.12629408E+03	-.00000000E-99	.75061614E+04	.33018300E+01
.36455565E+03	.25760895E+05	-.12543326E+03	-.00000000E-99
.75600799E+04	.00000000E-99	.00000000E-99	.00000000E-99
.00000000E-99	.00000000E-99	.00000000E-99	.38096687E+01
-.13156760E+01	.00000000E-99	-.12629408E+03	.11160542E+02
.59025372E+03	.33018300E+01	-.12699783E+01	-.00000000E-99
.12543326E+03	.00000000E-99	.00000000E-99	.00000000E-99
.00000000E-99	.00000000E-99	.00000000E-99	-.66962415E+01
.00000000E-99	.00000000E-99	.00000000E-99	-.00000000E-99
-.25400000E+02	-.00000000E-99	.59256844E+03	.53990843E+05
.00000000E-99	.00000000E-99	-.25400000E+02	-.00000000E-99
.00000000E-99	.00000000E-99	.00000000E-99	.28392180E+02
.00000000E-99	.00000000E-99	.00000000E-99	.12543326E+03
-.00000000E-99	.75600799E+04	.00000000E-99	.00000000E-99
.25533232E+05	-.12543326E+03	-.00000000E-99	.75600799E+04
.00000000E-99	.00000000E-99	.00000000E-99	-.65225299E+02
.00000000E-99	.00000000E-99	.00000000E-99	-.12699783E+01
.00000000E-99	-.12543326E+03	.11138937E+02	.59256844E+03
.00000000E-99	-.12699783E+01	-.00000000E-99	.12543326E+03
.00000000E-99	.00000000E-99	.00000000E-99	.24324772E+01
.00000000E-99	.00000000E-99	.00000000E-99	.00000000E-99
.00000000E-99	.00000000E-99	-.00000000E-99	-.25400000E+02
-.00000000E-99	.59025372E+03	.53784326E+05	-.36455565E+03
.00000000E-99	-.25400000E+02	-.00000000E-99	-.57448090E+02
.00000000E-99	.00000000E-99	.00000000E-99	.00000000E-99
.00000000E-99	.00000000E-99	.12543326E+03	-.00000000E-99
.75600799E+04	-.33018300E+01	-.36455565E+03	.25760895E+05
-.12629408E+03	-.00000000E-99	.75061614E+04	.70249819E+02
.00000000E-99	.00000000E-99	.00000000E-99	.00000000E-99
.00000000E-99	.00000000E-99	-.12699783E+01	.00000000E-99
-.12543326E+03	.11160542E+02	.59025372E+03	-.33018300E+01
-.13156760E+01	-.00000000E-99	.12629408E+03	.34613340E+00
.00000000E-99	.00000000E-99	.00000000E-99	.00000000E-99

TABLE A.3 (continued)

.000000000E-99	.000000000E-99	.000000000E-99	.000000000E-99
.000000000E-99	-.000000000E-99	-.254000000E+02	-.000000000E-99
.62175387E+03	.57124166E+05	.15400826E+04	-.41035754E+03
.000000000E-99	.000000000E-99	.000000000E-99	.000000000E-99
.000000000E-99	.000000000E-99	.000000000E-99	.000000000E-99
.000000000E-99	.12629408E+03	-.000000000E-99	.75061614E+04
-.11731337E+03	.15400826E+04	.19522989E+05	.50905068E+02
.000000000E-99	.000000000E-99	.000000000E-99	.000000000E-99
.000000000E-99	.000000000E-99	.000000000E-99	.000000000E-99
.000000000E-99	-.13156760E+01	.000000000E-99	-.12629408E+03
.10257644E+02	.62175387E+03	-.11731337E+03	-.66988605E+01

TABLE A.4 - OUTPUT OF PROGRAM NO. 3

SAMPLE DATA - SOUTH TRUSS, OCTOBER LOAD DISTRIBUTION, 30 TONS.

DETERMINANT = 2.6673449E+47

JOINT NUMBER	DELTA	ALPHA	BETA
2	-.019	-4.2752626E-04	6.0999643E-04
3	-.437	-1.9953873E-03	2.6128725E-03
4	.566	-5.6886206E-03	9.9489657E-04
5	.237	-3.6964109E-03	-3.4083734E-03
6	-.761	1.1568272E-03	-2.2793351E-03

APPENDIX B

LABORATORY INVESTIGATION OF A SINGLE CHANNEL
FLEXURAL MEMBER

In order to correctly evaluate the bending moments in the vertical members of the truss, it appeared from the 1959 data that some study should be made of the placement of the strain gauges on a channel cross-section. The results of this investigation are presented below.

Analysis. There are four possible sources of longitudinal strain in a rolled section: direct stress, flexure about two mutually perpendicular axes, and torsion. The first three of these are straightforward; however, some comment on torsion would seem desirable.

As an I-shaped or channel section twists, the flanges act in the manner of small beams, bending about an axis perpendicular to the face of the flange.⁽⁷⁾ The ensuing longitudinal strains are distributed as shown in Figure B.1*.

Considering now a channel section under the action of all four types of loading, the strains are as shown in Figure B.2, tension

*It is recognized that the strain at the heel of the channel is not identical to that at the toe. The results of the present investigation indicate, however, that no significant error is introduced by this approximation.

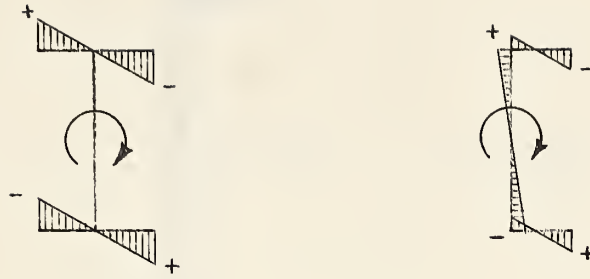


Figure B.1

positive. From Figure B.2, it is evident that there exist four unknown strains, suggesting that only four gauges might be required to

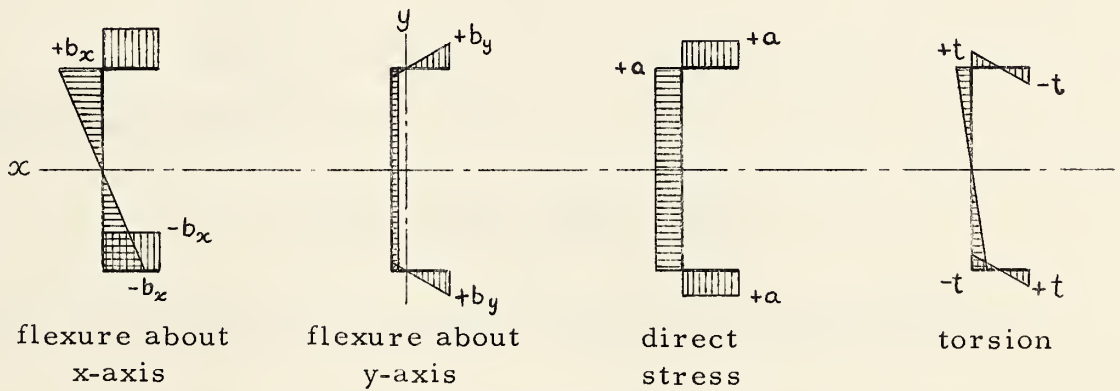


Figure B.2

eliminate unwanted effects. Suppose, therefore, that strains are measured at the points shown in Figure B.3. Designating the total longitudinal strains at these points by the circled numerals, and referring to Figure B.2, four simultaneous equations may be set up:

$$\textcircled{1} = a + b_x - 0.195 b_y + t$$

$$\textcircled{2} = a + b_x + b_y - t$$

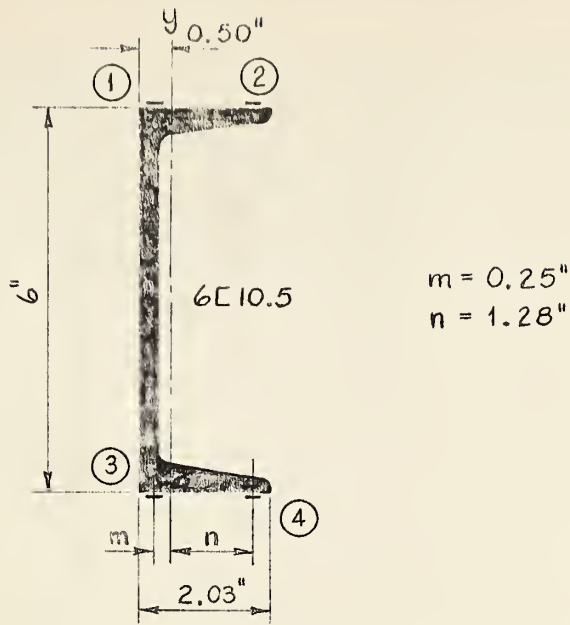


Figure B.3

$$\textcircled{3} = a - b_x - 0.195 b_y - t$$

$$\textcircled{4} = a - b_x + b_y + t$$

whence the required bending strain,

$$b_x = \frac{(\textcircled{1} + \textcircled{2}) - (\textcircled{3} + \textcircled{4})}{2}$$

having due regard for the algebraic sign of the measured strains.

Specimen. A six-inch standard channel weighing 10.5 pounds per foot was prepared as shown in Figure B.4. Baldwin SR-4 strain gauges were applied at two different stations and arranged on the cross-section as indicated.

Procedure. Two series of tests were carried out, one with the load applied on the flange one-half inch from the back of the channel and the other with the load applied one-and one-half inches from the

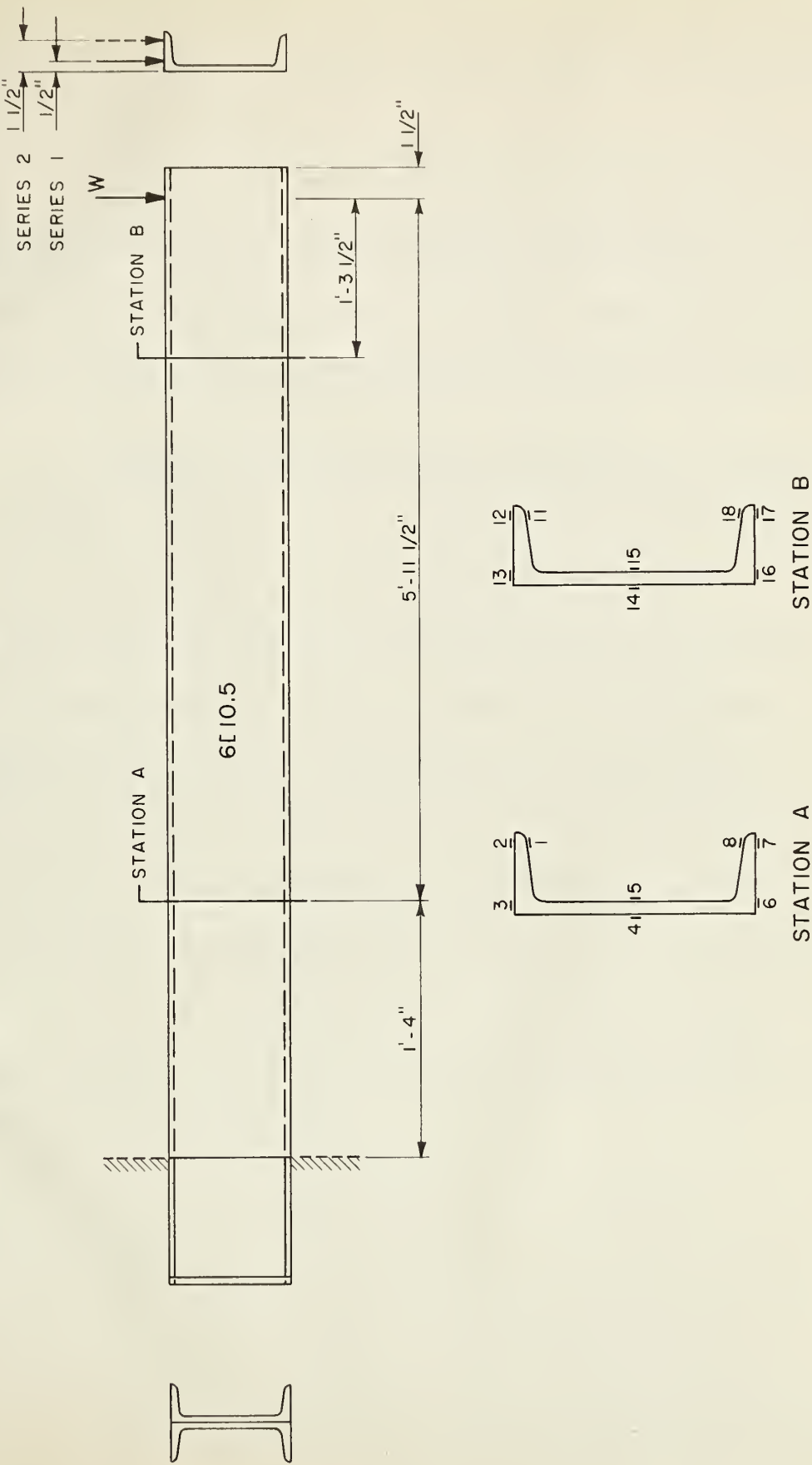


FIGURE B.4 - CHANNEL TEST SPECIMEN

back of the channel. Two tests were carried out in the first series, and one in the second.

The specimen was supported as a cantilever by clamping one end in the mechanical testing machine at the University of Alberta. Load was then applied by means of weights hung on the free end of the beam. Strains were recorded after each increment of loading.

Results. The experimental bending strains, computed as previously derived, are shown in Figure B.5, with the strains found by the simple flexure formula indicated by the solid lines. From this diagram it may be seen that the maximum error in the bending strain is approximately six per cent. A change in the eccentricity of the load appears to have made a slight difference, suggesting that the effects of torsional strains have not been entirely eliminated. This is to be expected, because, as explained in the footnote on page 166, the torsional strain distribution shown in Figure B.2 is somewhat in error. It was felt that the method outlined above for the reduction of strain readings gave results of such accuracy that a more involved analysis was not warranted.

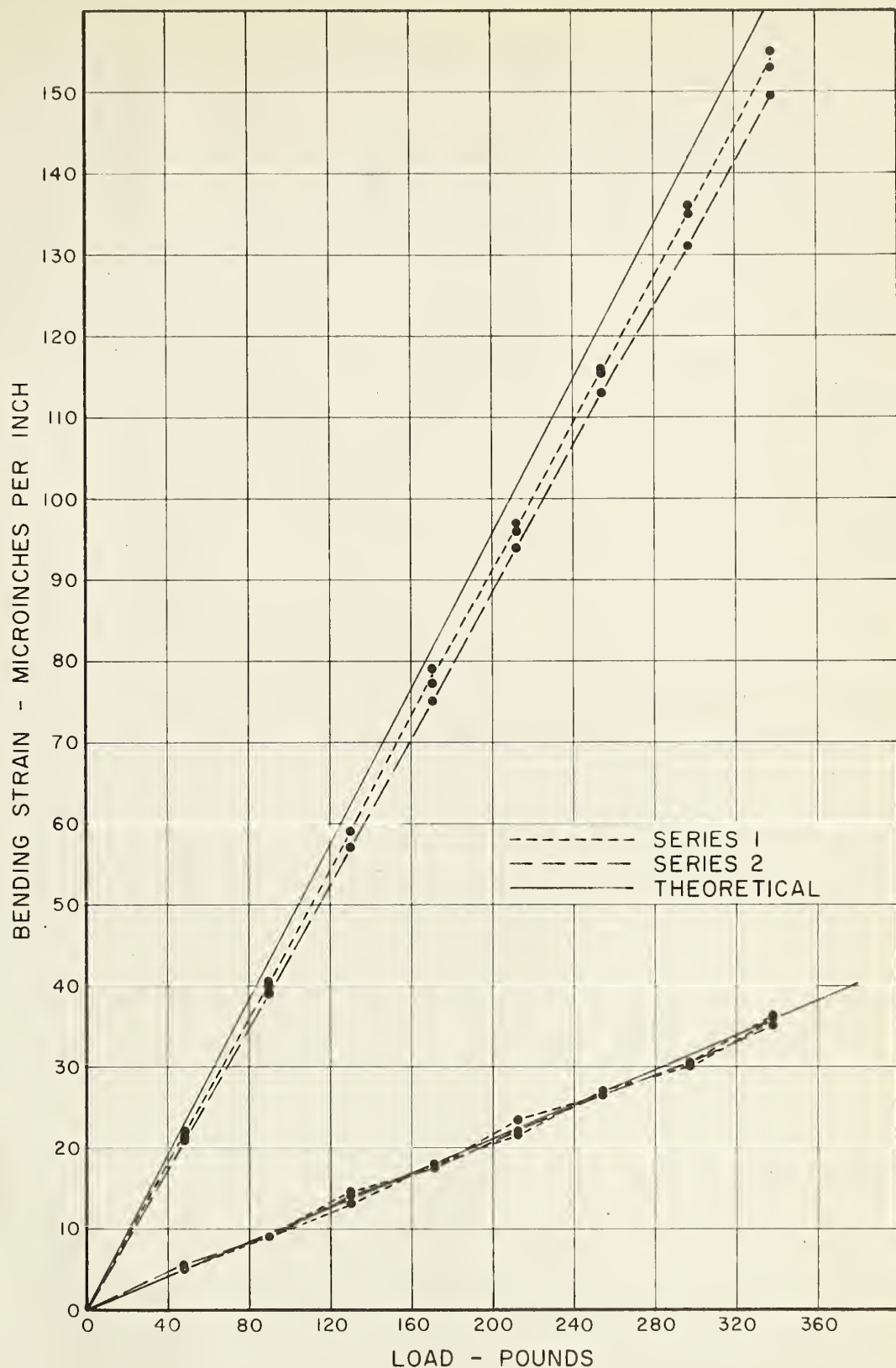


FIGURE B.5 - STRAINS DUE TO BENDING ABOUT THE MAJOR AXIS OF A SINGLE CHANNEL MEMBER

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